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Spatial modeling of atmospheric delays for InSAR and GPS sensors

R. Grebenitcharsky and R. Hanssen
Delft Institute for Earth Observation and Space Systems
Delft, the Netherlands

{r.grebenitcharsky}{r.f.hanssen}@lr.tudelft.nl

BACKGROUND

The high spatial resolution of InSAR data allows atmospheric delays to be spatially modeled by least-squares collocation (LSC). LSC requires a covariance model consistent with data. The spectral behavior of atmospheric delays derived by InSAR data allows the implementation of a stochastic model corresponding to the so-called Matérn class of covariance functions. The parameters of the power spectrum density function (variance, power-law exponent and frequency shift) can be estimated in the spectral domain and used for constructing the corresponding variance-covariance function in the spatial domain. Introducing an additional parameter (shift in frequency domain) provides more flexibility in modeling different weather conditions. Based on Matérn class covariance functions a new covariance function of atmospheric delays has been derived.

SPECTRAL REPRESENTATION OF PSD

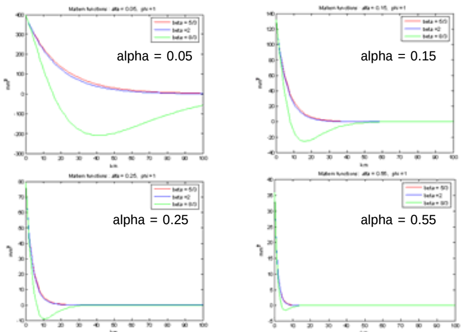
$$P(\omega) = \phi\left|\omega^2 + \alpha_\omega^2\right|^{-\beta/2}$$

SPATIAL REPRESENTATION – MATERN COVARIANCE FUNCTION

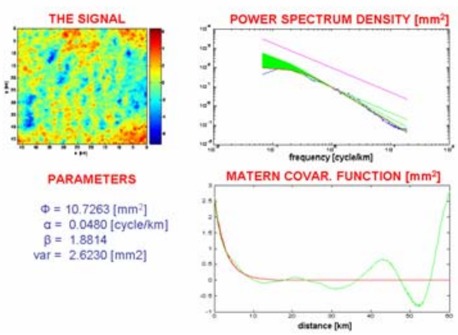
$$C(r) = C(r,\phi,\alpha_f,\beta) = \pi\phi\left\{\frac{\exp(-\alpha_f r)}{\alpha_f} + r - \frac{r^{(\beta-1)}}{\Gamma(\beta)/\sin\left(\frac{\beta-1}{2}\pi\right)}\right\}\exp(-\alpha_\omega r)$$

$$\text{where: } \alpha_\omega = 2\pi\alpha_f$$

SHAPE OF MATERN COVARIANCE FUNCTION



MATERN COVARIANCE FUNCTION OF REAL SIGNAL



If a spatially distributed trend is not presented in the signal the analytical Matérn class covariance function (red) approximates very well the empirical covariance function (green).

SIMULATED SIGNAL

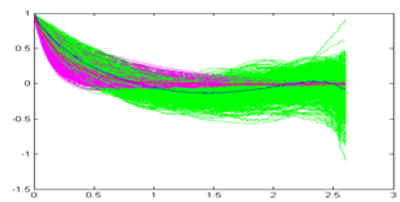
One atmospheric regime ($\beta = 8/3$)

Different weather conditions are simulated by changing the parameters of PDO. Three different weather conditions have been simulated for the regime between 0.25 km and 1.5 km. The power is kept constant and the coefficient *P* is varying for 3 weather conditions. The following table and graph show how the parameters are changing when the weather conditions are varying. It is obvious that both parameters are quite stable (the shape of all covariance functions in the graph remains the same). The main variation is due to the scaling factor. The results show that for 1000 samples for every weather condition the normalized empirical and analytical functions converge strongly to their average. It can be assumed that the average analytical covariance functions represent quite well the average empirical covariance functions in all weather conditions. Different weather conditions will influence only the scale (the variance) of the covariance functions. Due to the fact that the signal represents a stationary and isotropic stochastic process (without a deterministic component) the solution for all three parameters is very stable and converges to the values preliminary assigned.

Parameters	P=100000	P=1000000	P=10000000
α	2.33838	2.37061	2.36075
β	2.78307	2.78585	2.78360
var(mm^2)	3.3752	33.5934	329.3811
Φ (mm^2)	1898.0	16559.0	177691.5

Mixed atmospheric regime ($\beta_1 = 5/3,\ \beta_2 = 8/3$)

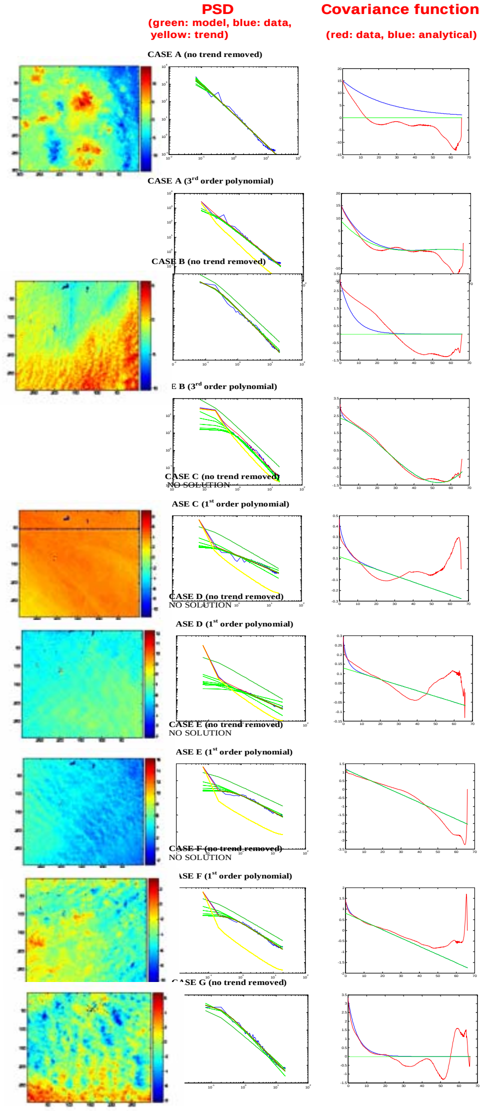
A simulation with a mixed atmospheric regime has been performed for different atmospheric conditions. Different values for *P* change only the scale of the covariance function. The solution for the covariance function parameters is stable and the analytical covariance function (magenta) converges to an average covariance function (red). The empirical covariance function (green) converges to its average (blue) one as well. One experiment with around 25000 simulations is shown below. It is visible that the average of the analytical model approximates quite well the average of the empirical covariance model. Those numerical simulations with a mixed atmospheric regime shows that if in the real data a trend is not present, the derived analytical covariance model is expected to approximate the empirical covariance function in a reasonable way.



REAL DATA EXPERIMENT

Different atmospheric conditions have been considered to test the compatibility between the analytical and empirical covariance functions. The presence of trend in the real signal causes instability in the determination of the parameters of the covariance function. To check if the existing trend is the reason for instable estimations of the parameters three different scenarios are considered. The parameters are determined after removing the spectrum of a linear, quadratic and third order polynomial trend from the spectrum of the signal. For different atmospheric conditions different order of the trend insures stable estimations of the parameters of Matern covariance function. The following table shows the order of the trend where the first stable estimation of the parameters appears and the parameters of the covariance function.

CASE	Order of trend	α	β	Φ
A	3	0.1101	1.7520	17.7095
B	3	0.7775	2.1609	9.4610
C	1	0.2564	1.3089	1.9612
D	1	0.3906	1.3515	0.7308
E	1	0.6267	2.0567	2.6586
F	1	0.4910	1.9861	6.1600
G	0	0.1763	18519	11.5358



CONCLUSIONS

• A new analytical covariance function corresponding to the spectral representation of Kolmogorov law has been derived, based on the Matern class of covariance functions. It will allow us to derive the parameters of the covariance function using the power spectrum density of the signal.
• The simulations with one atmospheric regime show that if a trend in the signal is not present, the estimations of the parameters are stable and converge to the initial parameters used by the simulation procedure.
• For real data the stability of the solution depends on the presence of a trend in the signal. The atmospheric signal can be assumed as stationary and isotropic all over the globe, but over a limited territory (e.g., one InSAR interferogram) an additional 'local trend' can be presented. In this case the effect of the trend on the spectrum of the signal needs to be extracted.
• For a SAR interferogram which will contain not only atmospheric delays a possible solution can be in the use of a local tropospheric delay model coming from permanent GPS stations.

TU Delft
Delft University of Technology