# On the treatment of radar interferometry in terms of geodetic adjustment and testing theory *


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Abstract: An empirical power law based covariance function is presented to model atmospheric error signal in radar interferograms. Results for eight different independent interferograms are evaluated, and the mathematical modeling of radar interferometric data is discussed.


## INTRODUCTION

In terms of geodetic adjustment and testing theory, radar interferometry has not yet evolved to maturity. This is caused by the fact that redundancy is often not ensured and a proper definition of the variance-covariance matrix of the observations has not been available. As a consequence, geophysical applications commonly focus on forward modeling, for example to duplicate the observed deformation pattern. While such an approach, apart from the apparent non-uniqueness, may be sufficient for improved geophysical understanding, it prohibits quantitative statements on precision and reliability.

In order to obtain such measures, a mathematical model is necessary which (i) relates the observations to the parameters, and (ii) models the variances and covariances of the observations. Regarding the first (functional) part of the model, every interferometric phase observation is related to at least five parameters: topographic height, surface deformation, atmospheric signal during the two acquisitions, and an integer unwrapping multiplicator. This means $n$ observations and $5n$ parameters. It is evidently impossible to solve this system. The standard approach to handle such a problem, for example for deformation monitoring, is to assume error-free phase unwrapping and eliminate the topography using a differential approach. Often it is also simply assumed that there is no atmospheric signal in the data, or clearly contaminated data are rejected manually. Regarding the second (stochastic) part of the model, the phase variances are usually obtained from coherence estimators, which gives a diagonal variance-covariance matrix, since the observations are assumed to be uncorrelated. In the following, we show that simple a-priori estimations of atmospheric variability can be used to improve the mathematical model.

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## MODEL

The observations $\varphi_k$ form a real stochastic vector of observations $\boldsymbol{\varphi} \in \mathbb{R}^n$, characterized by the first moment $E\{\boldsymbol{\varphi}\}$ and the second moment $D\{\boldsymbol{\varphi}\}$. After linearization, the relation between the observations and the unknown parameters can be written as a Gauss-Markoff model:

$$\begin{array}{ccccccc} \underset{n\times 1}{E\{\boldsymbol{\varphi}\}} & = & \underset{n\times m}{\mathbf{A}} & \underset{m\times 1}{\mathbf{x}}; & & & \\ \underset{n\times n}{D\{\boldsymbol{\varphi}\}} & = & \underset{n\times n}{\mathbf{C}_{\varphi}} & & = & \underset{1\times 1}{\sigma^2} & \underset{n\times n}{\mathbf{Q}_{\varphi}}, \end{array} \tag{1}$$

where $\mathbf{A}$ is the design matrix, $\mathbf{C}_{\varphi}$ is the variance-covariance matrix, $\mathbf{Q}_{\varphi} \in \mathbb{R}^{n\times n}$ the real positive-semidefinite $n\times n$ cofactor matrix, and $\sigma^2 \in \mathbb{R}^+$ the a priori variance factor. The vector of parameters $\mathbf{x} \in \mathbb{R}^m$ is assumed to be real and non-stochastic.

The first part of eq. (1) is commonly referred to as the functional model or model of observation equations, whereas the second part is known as the stochastic model. The functional model can also be written as

$$\boldsymbol{\varphi} = \mathbf{A}\mathbf{x} + \boldsymbol{\varepsilon}, \tag{2}$$

where $\boldsymbol{\varepsilon} \in \mathbb{R}^n$ reflects the errors of $\boldsymbol{\varphi}$ with $E\{\boldsymbol{\varepsilon}\} = 0$. This system is uniquely solvable if the design matrix $\mathbf{A}$ is of full rank, which implies that $n \geq m$. Moreover, $\mathbf{Q}_{\varphi}$ should be positive definite, since in that case the weight matrix of the observations, $\mathbf{Q}_{\varphi}^{-1}$, will exist and be positive definite as well The best linear unbiased estimator of the unknown parameters $\hat{\mathbf{x}}$ and the corresponding covariance matrix $\mathbf{C}_{\hat{\mathbf{x}}}$ is given by

$$\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{Q}_{\varphi}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Q}_{\varphi}^{-1}\boldsymbol{\varphi} \tag{3}$$

$$\mathbf{C}_{\hat{\mathbf{x}}} = \sigma^2(\mathbf{A}^T\mathbf{Q}_{\varphi}^{-1}\mathbf{A})^{-1} \tag{4}$$

The advantage of describing the inverse problem of parameter estimation in a systematic model such as the one outlined above is that different physical or geometrical aspects of the problem are reduced to relatively simple mathematical equations. Apart from providing a structural and perhaps elucidating approach, the formulation in a Gauss-Markoff model opens up a wealth of standard techniques for adjustment, and hypothesis testing. Moreover, such techniques give a quantitative measure of the accuracy and reliability of the estimated parameters.

## ATMOSPHERIC SIGNAL

What do we know about the atmospheric signal in interferograms? For a start, the relative character of an interferogram does not enable absolute signal delay measurements. Hence, we can demean the interferogram, which implies that the expectation value of the atmospheric signal for an arbitrary pixel will be zero. We also know that orbit errors can easily cause an almost linear trend over the whole interferogram. Such trends are usually hard to distinguish from atmospheric signal delay trends, and are usually eliminated using some kind of residual flattening, for example with tie-points. From experience, we know that the amount of atmospheric signal within an interferogram is highly variable in time: some interferograms seem to have no atmospheric influence at all, whereas others are contaminated significantly. Another specific characteristic is that the two states of the atmosphere are practically uncorrelated for an acquisition time interval of 1 day or more.

Several studies have shown that the predominant part of the atmospheric signal in interferograms is caused by the water vapor distribution in the lower troposphere. This is a consequence of the fact that for microwave frequencies, the water vapor variability dominates the variability of the refractive index. The water vapor distribution can be considered to be an approximate passive 'tracer' of the mechanical turbulence [4]. Unfortunately, this highly unpredictable behavior cannot be modeled in sufficient detail by other sensors, in contrast to other interferometric contributions such as topography. Therefore, in order to account for atmospheric signal, it can only be included in the stochastic part of the mathematical model.

The behavior of atmospheric signal in radar interferograms can be described using three interrelated measures: the power spectral density, the structure function, and the covariance function. The first is useful to recognize scaling properties in the data, the second is a useful quantitative expression for the variance of the difference in atmospheric delay between two points with distance $\rho$. The covariance function is necessary to construct the variance-covariance matrix $\mathbf{Q}_{\boldsymbol{\varphi}}$.

Based on the ideas of turbulence theory, it is expected that the signal has scaling properties. However, the scales observed in a $100 \times 100$ km interferogram cover three distinct regimes. First, for scales between 100-400 m and approx. 1.5 km, which are less than the tropospheric thickness, a one dimensional profile through the interferogram has a power exponent of $-8/3$, see [2, 3, 1]. Note that a large negative number implies a more smooth behavior. For larger scales, between approx. 1.5 km and 50-70 km, the turbulence is effectively two-dimensional since these scales are larger than the tropospheric thickness. In this regime, we observe an average power exponent of $-5/3$. As this number is less than $-8/3$, we expect the atmospheric signal to be more 'rough' for these scales. Although there is evidently atmospheric signal at scales which are even larger, for the stochastic modeling of atmospheric signal for InSAR we can safely ignore this, since it is outside the measurement capabilities of the system. However, based on the quality of the data, we usually observe a third distinct scaling region for scales between the pixel size (approx. 20 m) and 100-400 m. In this region we find an average power exponent of $-2/3$. These results can be written as:

$$S(\omega) = \begin{cases} C_o \omega^{-5/3} & \text{for } 1.5 \leq \omega^{-1} < 50 \text{ km}, \\ C_m \omega^{-8/3} & \text{for } 0.25 \leq \omega^{-1} < 1.5 \text{ km}, \\ C_i \omega^{-2/3} & \text{for } 0.02 \leq \omega^{-1} < 0.25 \text{ km}. \end{cases} \tag{5}$$

By defining $\omega$ as the wavenumber in cycles/km, and enforcing a continuous function, we can initialize this model by one parameter, by measuring the power spectral density at $\omega = 1$ cycle/km, which yields $C_m$. The other two coefficients are easily found to be $C_o = C_m \omega^{-1}$ for $\omega^{-1} = 1.5$ km and $C_i = C_m \omega^{-2}$ for $\omega^{-1} = 0.25$ km.

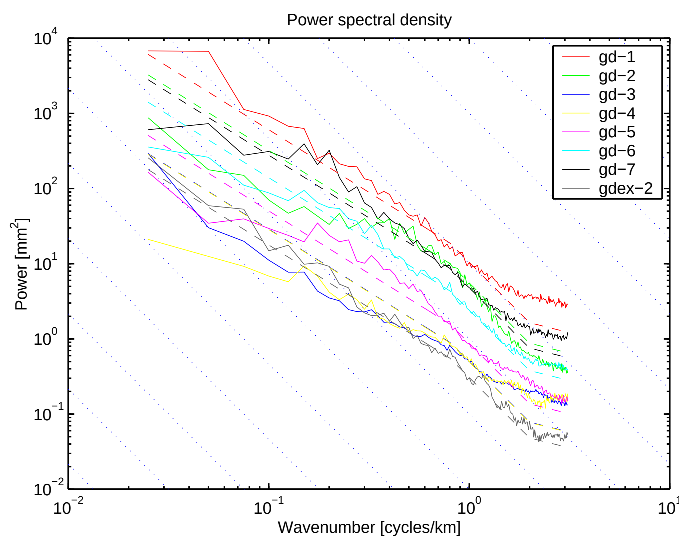


Figure 1: Power spectra for eight independent SAR interferograms, and the models defined by the power spectrum value at $\omega = 1$ cycle/km. The diagonal lines indicate a $-8/3$ power law.

Fig. 1 shows the empirical results of this approach, for eight independent interferograms, suffering from atmospheric signal varying from thunderstorms to calm weather. It is clear to see that the basic structure of all situations is similar, following the power law behavior described above. The diagonal lines indicate $-8/3$ power law values. Note that it is not possible, as some authors have proposed, to describe the scaling behavior with only one single power law. For wavelengths larger than 1.5 km, none of the signals follow the $-8/3$ lines. Although there is a strong similarity in shape, the difference between the absolute values of these curves varies two orders of magnitude. It is this variation that is important to consider while evaluating the quality of the interferometric data.

Fig. 2 shows the empirical covariance functions, derived from an inverse DFT of the power spectrum. Positive-definiteness of the covariance function is ensured by applying a Hamming weighting window for large distances, where it is obvious that all correlation will disappear.

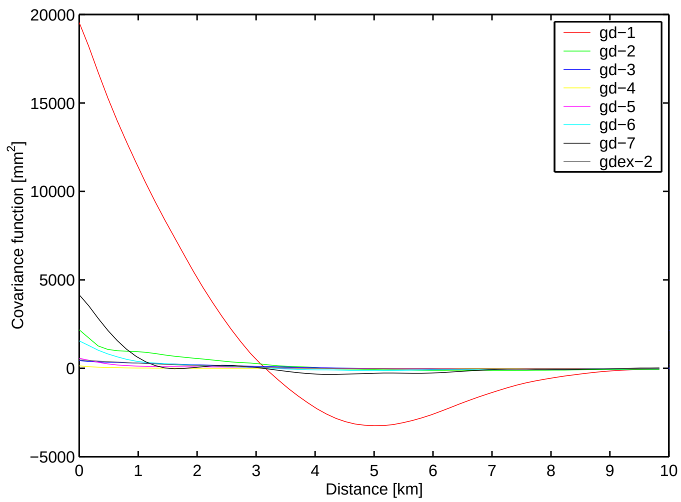


Figure 2: Covariance functions for eight atmospheric situations, derived from the power spectra models.

The covariance function can be used to fill the variance-covariance matrix $\mathbf{Q}_{\varphi}$, based on the distance between each pair of pixels. The diagonal elements consist of the model values and the phase variance derived from the coherence of the data.

## CONCLUSIONS

The definition of a stochastic model for radar interferometric measurements is a key element in describing the quality of the data and the derived parameters. Apart from the usual phase noise described by the interferometric coherence, the variances and covariances introduced by the atmosphere need to be included in this model as well. It is shown that, although the state of the atmosphere can be very different for interferograms, the shape of the power spectrum shows three distinct scaling regions, which are similar independent of the type of atmospheric behavior. Nevertheless, the absolute values of the power spectra differ two orders of magnitude.

Using the atmospheric model discussed in this paper, it is possible to quantitatively describe the covariance function of an interferogram using only one parameter. This parameter can be estimated from a small patch of the interferogram, with either even terrain or without deformation. This first order estimate may then be used to describe covariances in the whole interferogram. An alternative approach is the estimation of the initialization parameter using, e.g., GPS time series. In this way, a single GPS receiver in the interferogram area would be sufficient to quantify the covariances in the interferometric data. In the future, we will investigate these possibilities, and try to invert the variance-covariance matrices to adjust deformation models derived from series of interferograms.

## ACKNOWLEDGMENT

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