Phase Ambiguity Resolution For Stacked Radar Interferometric Data

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BIOGRAPHY

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ABSTRACT

Phase ambiguity resolution is one of the main problems in geodetic techniques involving electromagnetic phase measurements. For repeat-pass radar interferometry, the phase difference between two resolution cells in an interferogram usually reflects a topographic height difference, line-of-sight surface deformation, and atmospheric delay, or a combination of the three. Conventional techniques for two-dimensional phase ambiguity resolution, commonly referred to as ‘phase unwrapping’, apply path-following integration methods, using heuristic assumptions on the maximum phase gradient between adjacent resolution cells. If several SAR acquisitions are available, constraints can be introduced to estimate phase differences between two resolution cells, irrespective of the phase behavior of resolution cells in between these observations. This is advantageous, i.e., when areas of the data are very noisy and only a few coherent resolution cells can be identified. We can describe the interferometric phase observations as a linear function of topographic height (depending on the geometric satellite configuration), surface deformation (dependent of the time interval between the observations), and the integer ambiguity parameters, regarding the atmospheric signal as a component of the stochastic model. Hence, the problem is to resolve the real-valued unknowns and the integer ambiguities. In this paper it is demonstrated how the integer least-squares principle can be applied for this problem, and how a priori estimates on the success rates of the method can be given based on the satellite acquisition characteristics.

INTRODUCTION

Spaceborne synthetic aperture radar (SAR) interferometry has become an increasingly applied geodetic technique for topographic mapping, surface deformation monitoring, and atmospheric water vapor mapping, see e.g. Bamler and Hartl (1998); Hanssen (2001) for a general introduction of the technique. Interferometric radar data processing is an elaborate procedure, involving the focusing of the raw data to complex SAR images, the alignment and complex multiplication of two images to form an interferogram, and several processing steps to convert the phase of the interferogram to, e.g., a geocoded digital elevation model or surface deformation map. One important step in this procedure is the so-called *phase unwrapping*, or integer ambiguity resolution, in which relative phase observations, measured modulo 2π, are converted to absolute phases.

The problem of 2D phase unwrapping in radar interferometry relies heavily on assumptions on (i) the maximum phase gradient and (ii) the preservation of coherence between adjacent observations (pixels or resolution cells). Steep topographic slopes, combined with large perpendicular baselines between the satellites, or high rates of surface deformation fail to fulfill the first assumption while spatially varying degrees of temporal decorrelation (changes in the backscatter characteristics of the terrain) hamper the second one. As a result, there are many cases in which conventional phase unwrapping is not possible or provides erroneous or non-unique results, making the development of alternative methods necessary.

For a single interferogram, we can write the generalized functional model of linear(ized) observation equations as

$$
\begin{aligned}
W(\phi_p - \phi_q) &= \phi_{p,q} \\
&= A\begin{bmatrix} H_{p,q} & D_{p,q} & k_{p,q} \end{bmatrix}^T + e_a + e_n \\
A &= 2\pi \begin{bmatrix} \kappa B'_\perp & \kappa & -1 \end{bmatrix} \\
\kappa &= 2/\lambda \\
B'_\perp &= -B_\perp/(R\sin\theta^o)
\end{aligned}
\tag{1}
$$

where

- $\phi_{p,q}$ is the observed phase difference between two pixels p and q,
- $W(\cdot)$ is the wrapping operator, with $W(\phi) = \mathrm{mod}(\phi+\pi, 2\pi) - \pi$,
- A is the design matrix, with $B_\perp$ the normal component of the interferometric baseline, R the slant range and θ^o the reference incidence angle,
- $H_{p,q} \in \mathbb{R}$ is the unknown topographic height difference,
- $D_{p,q} \in \mathbb{R}$ is the relative surface deformation, and
- $k_{p,q} \in \mathbb{Z}$ is the integer ambiguity.

We omit the subscripts p, q from now for brevity, stressing that we are involved with interferometric phase *differences*. The error term e_a expresses the atmospheric error with $E\{e_a\} = 0$ and $D\{e_a\} = f(l)$, where l is the distance between the two pixels. It is clear that in this formulation the design matrix A has a rank defect of 2, and even if either H or D were known, the integer ambiguity still results in a rank defect. Since a single radar interferogram does not introduce any redundancy to solve this problem, assumptions for phase unwrapping as mentioned above are unavoidable, evidently leading to errors or manual corrections in less than ideal situations.

It has been shown (Ferretti *et al.*, 1996, 2001) that the availability of several radar acquisitions over an area can be used to add observations to the problem of eq. (1). In fact, the availability of N radar acquisitions leads to $(N^2 - N)/2$ possible interferometric combinations, of which $N - 1$ are uncorrelated. Such a series of interferograms, all aligned to the same reference grid, is referred to as a stack of interferograms. Nevertheless, the integer ambiguity problem still needs to be resolved. In the following we describe how a procedure labeled *integer least-squares*, developed by Teunissen (1993) can be used for this type of problem. We limit ourselves for this first evaluation to the problem of linear deformation rate estimation, assuming that topographic phase components are eliminated. Extension of the model to more unknowns is relatively straightforward.

MODEL

Suppose M radar acquisitions are available over an area showing linear deformation, e.g., subsidence due to the extraction of hydrocarbons. Choosing a suitable reference image at time t_i it is possible to create interferograms with respect to this reference image. Using a backscatter amplitude based selection mechanism, see Ferretti *et al.* (2001) we can detect randomly distributed single pixels which behave as *permanent scatterers*: these have a high phase accuracy ($\sigma_\varphi \leq 5$ degrees) and behave systematically in time. The phase observations for times $[t_1..t_{i-1}, t_{i+1}..t_n]$ between every pair of permanent scatterers can be written in a linear model of observation equations as:

$$E\left\{\begin{bmatrix}\varphi_1\\ \vdots\\ \varphi_{i-1}\\ \varphi_{i+1}\\ \vdots\\ \varphi_n\\ v_{\text{ps}}\end{bmatrix}\right\} = \begin{bmatrix} k\,\Delta t_1 & -2\pi & & \\ \vdots & & & \\ k\,\Delta t_{i-1} & & \ddots & \\ k\,\Delta t_{i+1} & & & \\ \vdots & & & \\ k\,\Delta t_n & & & -2\pi \\ 1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} v\\ w_1\\ \vdots\\ w_{i-1}\\ w_{i+1}\\ \vdots\\ w_n\end{bmatrix} \tag{2}$$

where φ_j reflects the observed interferometric phase difference at $t = t_j$ between two permanent scatterers. Parameters to be estimated are $v \in \mathbb{R}$, the deformation rate (in m/y) and $w_j \in \mathbb{Z}$ with $j = 1 \ldots n \backslash i$, the integer ambiguities for every observation. Since the design matrix has a rank defect, a pseudo observation v_{ps} is added to the observation vector. Factor $k = -4\pi/\lambda$ and $\Delta t_j = t_i - t_j$, with t expressed in years. The variance of the interferometric phase is derived from the signal-to-noise ratio of the amplitudes of the point scatterers (Ferretti *et al.*, 2001).

The solution of this problem can be found using the integer least-squares methodology, which can be divided into three different steps. First, the integer constraints on the unknown ambiguities w_j are disregarded, assuming $w_j \in \mathbb{R}$. Performing a standard least-squares adjustment, we find the so called *float solution* giving the real-valued estimates of v and w, together with their variance-covariance matrix

$$\begin{bmatrix}\hat{v}\\ \hat{w}\end{bmatrix}; \quad \begin{bmatrix} Q_{\hat{v}} & Q_{\hat{v}\hat{w}} \\ Q_{\hat{w}\hat{v}} & Q_{\hat{w}} \end{bmatrix}. \tag{3}$$

In the second step, the float ambiguity estimate $\hat{w}$ is used to compute the corresponding integer estimate, denoted by $\check{w}$. This is equivalent to solving the minimization problem

$$\min_{w \in \mathbb{Z}^{n-1}} (\hat{w} - w)^T Q_{\hat{w}}^{-1} (\hat{w} - w) \to \check{w} \tag{4}$$

in the metric defined by the variance-covariance matrix $Q_{\hat{w}}$. If $Q_{\hat{w}}$ is diagonal, the solution of eq. (4) can be simply obtained by rounding the float solution to the nearest integer solution. However, often the float ambiguities are correlated resulting in an elongated, $n - 1$ dimensional error ellipse. As a consequence, simple rounding of the float solution may not result in the best estimate for the integer ambiguity. A decorrelation procedure, termed least-squares ambiguity decorrelation adjustment (LAMBDA) has been developed as a rigorous and efficient way to compute the integer ambiguities, see e.g., Teunissen (1993); de Jonge and Tiberius (1996). Finally, once the integer solution is computed, it is used in the third step to correct the *float* estimate of v to find the *fixed* solution, using

$$\check{v} = \hat{v} - Q_{\hat{v}\hat{w}} Q_{\hat{w}}^{-1} (\hat{w} - \check{w}) \tag{5}$$

hence, the unwrapped phase observations can be written as $\varphi + 2\pi\check{w}$.

EXAMPLE

Figure 1 shows a simulation of ambiguity resolution between three permanent scatterers in a stack of 12 radar acquisitions, acquired irregularly over a period of 6 years. The three permanent scatterers form a triangle and the phase differences of the three sides of the triangle are shown in the three rows of fig. 1.

The left column of plots show the simulated data as red crosses. The simulated data are normally distributed around their 'true' values, with $\sigma_\varphi = 5$ degrees, which is a realistic value for data selected with an amplitude SNR of 4 or more (Ferretti *et al.*, 2001). The data used as input

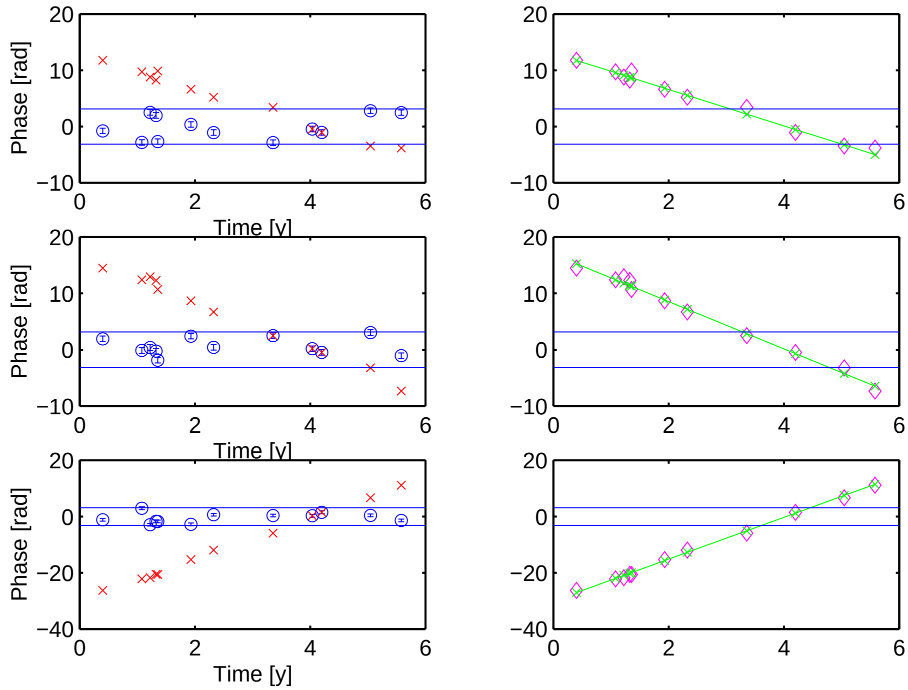

Fig. 1. Simulation of integer least-squares ambiguity resolution for three permanent scatterers, visible in 12 radar acquisitions over a period of 6 years. The left column shows the absolute and relative (wrapped) phases for the three sides connecting the three pixels. The right column shows the result of the integer ambiguity resolution (red diamonds) and the entire adjustment (green crosses and line).

in the integer least-squares simulation are 'wrapped' to the $[-\pi, \pi)$ interval, indicated by the two horizontal blue lines. The wrapped data are indicated by the blue circles. Thus, the challenge of the problem is to derive the positions of the red crosses from the blue circles.

In the right column of plots, the results of the estimation are given. Since the 12 radar acquisitions result in 11 interferograms, we use one acquisition as reference (master). Here, the reference acquisition at $t \approx 4$ years is used: it is visible in the left column, but not in the right column. All 'unwrapped' results are vertically shifted with respect to this reference point. The red diamonds show the unwrapped data while the green crosses, connected by the green line to indicate the constant deformation rate, show the results of the total adjustment. A pseudo observation, see eq. (2), is introduced in order to give the design matrix full rank. Since the deformation can be positive as well as negative, we used zero as pseudo observation, with a standard deviation of 10 rad/year to quantify our knowledge, or better, uncertainty, of the expected deformation rate.

Comparing the simulated absolute data in the left column with the results of the integer adjustment in the right column shows an excellent agreement. In the following paragraph, we show that it is possible to determine the probability of a successful adjustment based on the accuracy of the data and the number and temporal distribution of the acquisitions.

SUCCESS RATE

Several possibilities for integer ambiguity estimation are available. Simple rounding does not take the stochastic part of the observations into account, in contrast to bootstrapping (a form of sequential conditional least-squares) and integer least-squares. Teunissen (1999) has shown that the integer least-squares estimator is optimal in the sense that it maximizes the probability of correct integer ambiguity estimation. Unfortunately this probability, known as the success rate, is difficult to compute numerically for the integer least-squares estimator. For the bootstrap estimator, however, it is relatively easy to compute. Since the integer least-squares method outperforms the bootstrap method, the bootstrapped success rate can be regarded as a lower bound of the actual success rate applying the integer least-squares method (Teunissen, 1999). Consequently, it is possible to verify the potential feasibility of applying integer least-squares ambiguity resolution on a given set of data before any actual computations.

The success-rate is defined as $P(\check{w} = w)$, and can be derived from $Q_{\hat{w}}$, by decomposing $Q_{\hat{w}} = L\,D\,L^T$. It can be written as

$$0 \leq P(\check{w} = w) = \prod_{i=1}^{n} \left[2\Phi(\frac{1}{2\sigma_{i|I}}) - 1 \right] \leq 1 \qquad (6)$$

where $\sigma^2_{i|I}$ are the conditional variances, which are the diagonal elements of D. The function $\Phi(x)$ is defined as:

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\{-\frac{1}{2}v^2\} dv \qquad (7)$$

The variance-covariance matrix of the float solution $Q_{\hat{w}}$, see eq. (3), necessary for computing the success rates, can be constructed from the design matrix, see eq. (2), and the variance-covariance matrix of the observations Q_y. Hence, knowing the satellite acquisition times and the a priori variances of the observations and pseudo-observations, we can decide whether integer least-squares estimation is feasible and reliable for a specific application.

In fig. 2, the a priori computed success rates are plotted against the number of satellite acquisitions. Since the success rate is not only dependent of the number of acquisitions but also on their temporal distribution, every plotted value is the average of 100 simulations with random temporal distributions. Based on these values $\sigma_{v_{\mathrm{ps}}} = 10$

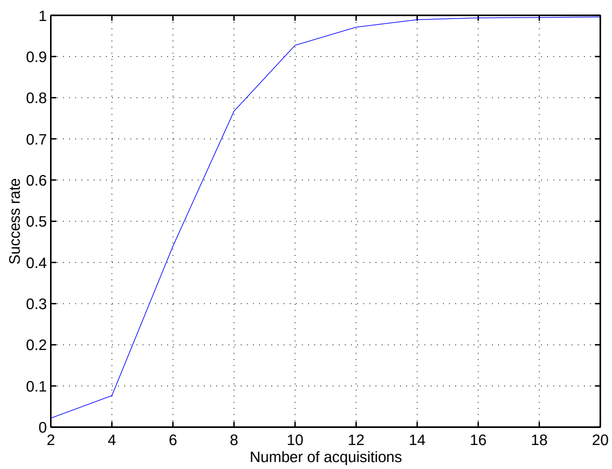

Fig. 2. The probability of successful ambiguity resolution, or *success rate*, expressed as a function of the number of acquisitions, randomly distributed over a period of 6 years, using fixed phase accuracies and pseudo observations.

rad/y) it is evident that 12 or 14 acquisitions are sufficient to resolve the ambiguities successfully in most cases. Obviously, in practice the actual distribution of the radar acquisitions needs to be applied for an optimal evaluation of the feasibility of the technique.

CONCLUSIONS

We have shown that the phase ambiguity resolution problem for stacked radar interferometric data can be formulated as an integer least-squares problem. This formulation provides a rigorous and efficient alternative for estimating parameters related to surface deformation or topography from series of radar acquisitions. An important practical advantage of ambiguity resolution using integer least-squares estimation is that an a priori estimate of (a lower bound of) the success rate can be derived, yielding a quantitative estimate of the feasibility of the method. Future investigations will focus on extending the model for batch estimation of groups of points, including topography estimation, evaluating the stochastic model, and applying the method to practical problems.

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