Modeling and spatial interpolation of tropospheric signal delay for space−geodetic observations based on GPS time series analysis

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Introduction

Space-geodetic observations such as VLBI, GPS, and InSAR are affected by tropospheric signal delay. The variability of water vapor, pressure, and temperature, both spatially and temporally, results in a tropospheric error signal that is correlated in time and space. This correlation mainly results from the turbulence characteristics of the tropospheric boundary layer and its dimensions. Over relatively large spatial scales, larger than 10 km, GPS networks and VLBI observations have been able to describe such spatial correlation characteristics, leading to a power-law approximation of the signal. Radar interferometry (InSAR) provides estimation of spatial correlation on small scales of 50 m–50 km. Unfortunately, InSAR observations are currently not available on a regular basis to serve as quantification of the tropospheric error signal. Here we investigate whether GPS time series, combined with Taylor's assumption of a 'frozen atmosphere' and readily available meteorological observations can serve as a parameterization of spatial tropospheric delay variability.

Spectral estimation from radar interferometry

Fig. 1 shows the one-dimensional power spectra of eight atmospheric situations, for spatial scales between 300 m and 40 km. The dotted lines indicate $-5/3$ and $-8/3$ power law slopes. It can be observed that the data can be coarsely related to a $-5/3$-slope for spatial scales larger than 3 or 4 km, a $-8/3$-slope for scales between

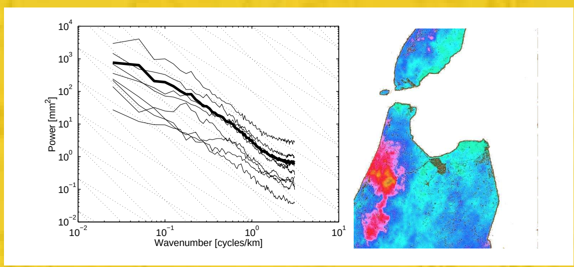

Figure 1: Tropospheric power spectra derived from radar interferometry. The bold line indicates the mean of the eight realizations. Right; example of water vapor distribution from InSAR.

250 m and 3 or 4 km, and a flattened behavior for smaller scales, probably due to observational noise. The power law model can be written as

$$P_d(k) = P_0 \cdot (k/k_0)^{-\beta}, \qquad (1)$$

with $\beta = 5/3$ and $8/3$ for $0.025 < k < 0.3$ cycles/km and $0.3 < k < 4$ cycles/km, respectively, and $k_0 = 1$ cycle/km. This model leaves initialization constant P_0 as single parameter to describe the atmospheric variability. Its apparent variation of two orders of magnitude is mainly caused by variability in the weather situation, i.e., degree of convection, wind shear, stability, etc. Hence, a stochastic description of the weather situation using the P_0 parameter would be valuable for error estimation in geodetic methods.

GPS time series

GPS delay time series can be converted to spatial delay measures using the following assumptions. First, it is assumed that within the extent of the time series the troposphere is 'frozen', that is, the refractivity distribution remains unchanged and drifts as a whole over the GPS receiver, with a wind speed constant in time and over altitude. Second, the time series observations are real zenith delays, preventing correlation between subsequent measurements. If these assumptions hold, conversion from temporal to spatial series is simply achieved by

$$d[t_i] \rightarrow d[w\,t_i] = d[x_i], \quad \text{with}$$
$$t_i = t_0 + i\,\Delta t; \quad x_i = x_0 + i\,\Delta x$$

where d is the wet delay, Δt is the sampling interval, w the wind speed, and $\Delta x = w\,\Delta t$ the corresponding spatial sampling. Fig. 2 shows a sketch of the signal time-space conversion and its effect on the power spectrum estimation.

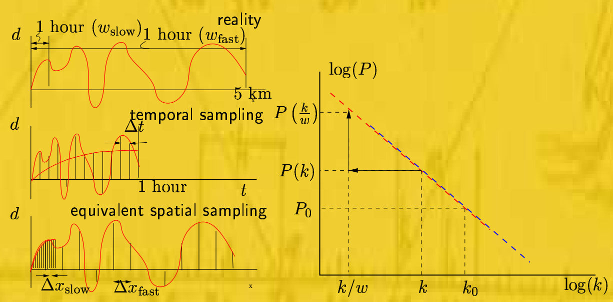

Figure 2: Principle of time-space conversion using wind speed and its consequence for the spectral estimation

Estimating wind speed

Evidently, the assumption of a wind speed that is constant over time and altitude is incorrect. However, the requirements for application of the time-space conversion of the delay signal can be relaxed by using an *effective* wind speed which is approximately constant at the altitude at which the variability in refractivity is maximal, and that changes in wind speed at that altitude in time are limited within the time of spectral estimation.
To investigate the consequences and limitations posed by this assumption, the mean difference of the refractivity over an interval of 6 hours is shown in fig. 3a. These data are obtained from one year radiosonde observations in the Netherlands. It is

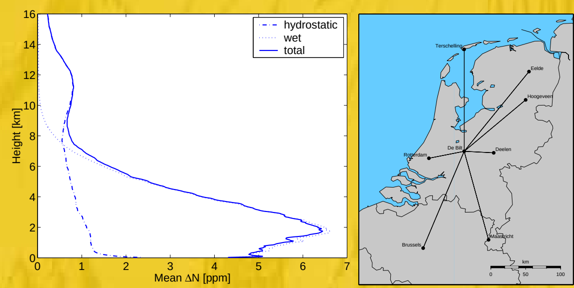

Figure 3: ***a)*** Mean difference in refractivity over a 6 hour period, using one year of radiosonde data. ***b)*** Location of the pressure measurents needed for the geostrophic wind estimation.

clear that the main short-term variability occurs at an altitude of approximately 2 km. There are two methods for estimating wind speed at altitude based on surface observations. (i) Using surface wind observations, a logarithmic increase with altitude is predicted using the model:

$$w(h) = w(h_{\text{ref}}) \frac{\ln(h/h_0)}{\ln(h_{\text{ref}}/h_0)}, \qquad (2)$$

where h_{ref} is the reference altitude (10 m), 0.2 mm$< h_0 <$ 2 m is the roughness length. Although this model is validated for $0 < h < 100$ m, we 'misuse' it to an altitude of 2 km. (ii) Using surface pressure observations, the geostrophic wind at altitude can be estimated for heights between 1.5 and 2 km:

$$w(1.5 < h < 2\text{km}) = \frac{\Delta P \Delta r}{\rho_m f_c |\Delta x||\Delta y|}, \qquad (3)$$

where ΔP, pressure gradient; Δr, spatial separation; ρ_m, density of moist air; f_c, Coriolis force; Δx and Δy, east and north separation. Both methods have been empirically tested using all radiosonde observations of one year as groundtruth.

True wind speed (2 km)	11.95 m/s	
Geostr.wind speed (2 km)	12.60 m/s	± 6.74
Logarith.wind speed (2 km)	8.81 m/s	± 5.00

Geostrophic estimation appears to be the best approximation, although the 1σ standard deviation shows a rather large dispersion.

Results

GPS time series of Delft, using a 30 s sampling rate, averaged and Kalman filtered to 6 min intervals, are converted to spatial delay with wind speeds derived using the geostrophic estimation. Fig. 3b shows the distribution of the pressure measurements. Fig. 4 shows the mean spatial power spectra obtained over a period of three years.

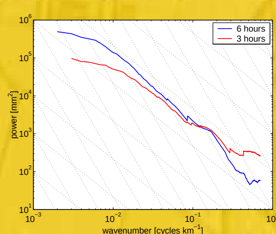

Figure 4: Three year mean spatial power spectra derived from GPS time series and wind speed, for estimation periods of 3 (red) and 6 (blue) hours. Diagonal lines indicate $-5/3$ and $-11/3$ slopes. The large slopes are mainly caused by the spectral signature of the Kalman filter, which was inseparable from the GPS data.

Discussion

Comparison between the radar and the GPS derived spectra shows for $k_0 = 0.1$ km^{-1} a P_0 of about 200 mm^2 and 2000 mm^2, respectively. Differences can be explained by the effect of the Kalman filter on the GPS data, errors in the estimated wind speed, and the atmospheric superposition in the interferogram. Future study will address these issues and comment on feasible accuracy ranges. Optimal estimation methods for single 3-6 hour spectra will be derived, leading to an estimation of the P_0 parameter.