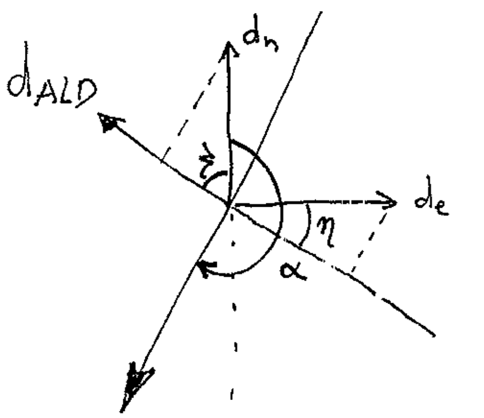
Decomposition of displacement vector $[d_u, d_n, d_e]$ in range and azimuth offsets for ascending and descending orbits

STEP (1): decomposition of d_n, d_e to horizontal azimuth look direction

DESCENDING

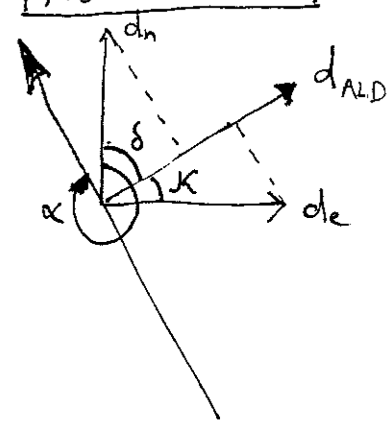


$$\xi = 2\pi - \alpha - \frac{\pi}{2} = \frac{3\pi}{2} - \alpha$$

$$\eta = \alpha - \pi$$

$$d_{ALD} = +d_n \cos\xi - d_e \cos\eta$$

ASCENDING



$$\delta = \alpha - \frac{3\pi}{2} = -\xi$$

$$K = 2\pi - \alpha = \pi - \eta$$

$$\begin{aligned} d_{ALD} &= +d_n \cos\delta + d_e \cos K \\ &= +d_n \cos -\xi + d_e \cos(\pi - \eta) \\ &= +d_n \cos\xi - d_e \cos\eta \end{aligned}$$

$$\boxed{\begin{aligned} \cos\eta &= \cos(\alpha - \pi) \\ &= -\cos\alpha \end{aligned}}$$

$$\begin{aligned} d_{ALD} &= +d_n \cos\left(\frac{3\pi}{2} - \alpha\right) + d_e \cos\alpha \\ &= +d_n \sin\alpha + d_e \cos\alpha \end{aligned}$$

α = heading satellite track w.r.t. north,

ALD = azimuth look direction

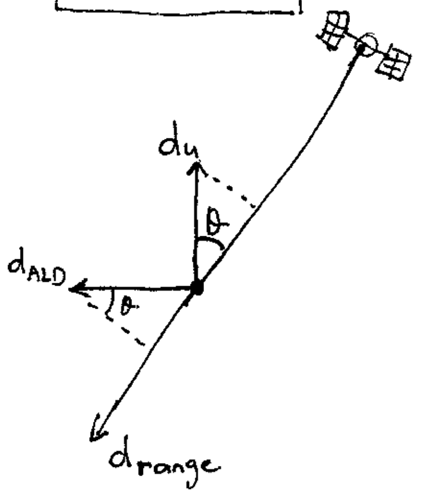
d_n = displacement vector component to north (northbound is positive)

d_e = " " " " east (eastbound is positive)

d_u = " " " vertical up (upward is positive)

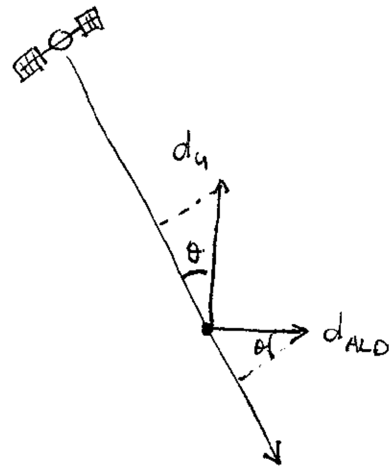
STEP 2 : composition of horizontal (ALD) displacement and vertical (d_u) displacement to slant range displacement or offset.

DESCENDING



$$d_{range} = -d_u \cos\theta + d_{ALD} \sin\theta$$

ASCENDING



$$d_{range} = -d_u \cos\theta + d_{ALD} \sin\theta$$

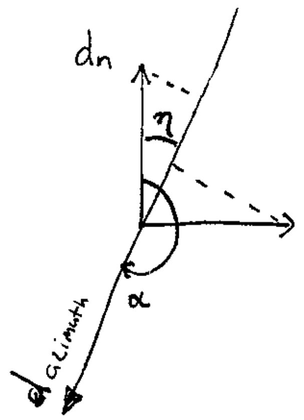
$$d_{range} = -d_u \cos\theta + (d_n \sin\alpha + d_e \cos\alpha) \sin\theta$$

d_{range} = displacement vector in radar line-of-sight (range increase is positive)

STEP 3 Azimuth offset (not for interferometric phase, only for speckle tracking / offset of pixels in coregistration procedure)

Composition of horizontal (d_n, d_e) displacement vectors to azimuth offset direction.

DESCENDING



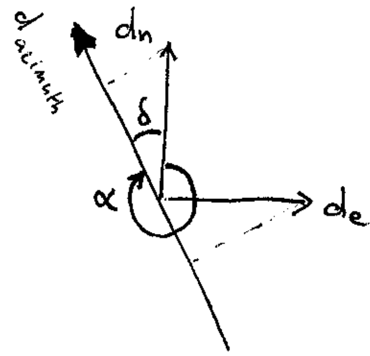
$$\eta = \alpha - \pi$$

$$d_{azi} = -d_n \cos\eta - d_e \sin\eta$$

$$\boxed{\cos\eta = \cos(\alpha - \pi) = -\cos\alpha} \quad \boxed{\sin\eta = \sin(\alpha - \pi) = -\sin\alpha}$$

$$d_{azi} = d_n \cos\alpha + d_e \sin\alpha$$

ASCENDING



$$\delta = 2\pi - \alpha = \pi - \eta$$

$$d_{azi} = +d_n \cos\delta - d_e \sin\delta$$
$$= +d_n \cos(\pi - \eta) - d_e \sin(\pi - \eta)$$
$$= +d_n \cos(2\pi - \alpha) - d_e \sin(2\pi - \alpha)$$
$$= d_n \cos\alpha + d_e \sin\alpha$$

$$d_{azi} = d_n \cos\alpha + d_e \sin\alpha$$

d_{azi} = azimuth offset vector (increasing azimuth is positive)

Functional model for estimating displacement vector $[d_u, d_n, d_e]$ from range and azimuth offset vectors from descending and ascending orbits.

Observations

- d_{rd} : displacement vector in range, descending orbit
- d_{ad} : " " " azimuth, " "
- d_{ra} : " " " range, ascending orbit
- d_{aa} : " " " azimuth, " "

Unknown parameters:

- d_u : displacement vector up
- d_n : " " north
- d_e " " east

Given:

- α_d : heading descending orbit
- α_a : " ascending orbit
- θ : local incidence angle

$$E\left\{\begin{bmatrix} d_{rd} \\ d_{ad} \\ d_{ra} \\ d_{aa} \end{bmatrix}\right\} = \begin{bmatrix} -\cos\theta & \sin\theta\,\sin\alpha_d & \sin\theta\,\cos\alpha_d \\ 0 & \cos\alpha_d & \sin\alpha_d \\ -\cos\theta & \sin\theta\,\sin\alpha_a & \sin\theta\,\cos\alpha_a \\ 0 & \cos\alpha_a & \sin\alpha_a \end{bmatrix} \begin{bmatrix} d_u \\ d_n \\ d_e \end{bmatrix};$$

$$D\left\{\begin{bmatrix} d_{rd} \\ d_{ad} \\ d_{ra} \\ d_{aa} \end{bmatrix}\right\} = \begin{bmatrix} \sigma^2_{d_{rd}} & & & 0 \\ & \sigma^2_{d_{ad}} & & \\ & & \sigma^2_{d_{ra}} & \\ 0 & & & \sigma^2_{d_{aa}} \end{bmatrix} = Q_d = \sigma^2_{\text{offset}} \begin{bmatrix} 5 & & & \\ & 1 & & \\ & & 5 & \\ & & & 1 \end{bmatrix}$$

For $\theta \approx 23^\circ$; $\alpha_d \approx 188^\circ$; $\alpha_a \approx 352^\circ$

$\sin\theta \approx 0{,}39$

$\cos\theta \approx 0{,}92$

$\sin\alpha_d \approx -0{,}14$

$\cos\alpha_d \approx -0{,}99$

$\sin\alpha_a \approx -0{,}14$

$\cos\alpha_a \approx +0{,}99$

$$A = \begin{bmatrix} -0{,}92 & -0{,}05 & -0{,}39 \\ 0 & -0{,}99 & -0{,}14 \\ -0{,}92 & -0{,}05 & +0{,}39 \\ 0 & +0{,}99 & -0{,}14 \end{bmatrix}$$

$$[A^T A]^{-1} A^T = \begin{bmatrix} -0{,}54 & 0{,}02 & -0{,}54 & -0{,}03 \\ 0 & -0{,}51 & 0 & 0{,}51 \\ -1{,}14 & -0{,}41 & 1{,}14 & -0{,}41 \end{bmatrix}$$

$$(A^T Q_y^{-1} A)^{-1} A^T Q_y^{-1} = \begin{bmatrix} -0{,}54 & 0{,}03 & -0{,}54 & -0{,}03 \\ 0 & -0{,}51 & 0 & 0{,}51 \\ -0{,}78 & -1{,}40 & 0{,}78 & -1{,}40 \end{bmatrix}$$

$$Q_y^{-1} = \begin{bmatrix} 5 & & & \\ & 1 & & \\ & & 5 & \\ & & & 1 \end{bmatrix}^{-1}$$