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An empirical model for the assessment of DEM accuracy degradation due to vertical atmospheric stratification

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ABSTRACT

This paper reports on the implications of vertical refractivity profiles in the troposphere on the quality of Digital Elevation Models and surface deformation maps derived from synthetic aperture radar (SAR) interferometry. A representative set of 1460 radiosondes acquired over one year in the Netherlands is used to obtain statistics for the differential delays between the two acquisition dates, and apply these to simulate 1-day and $k \times 35$ day intervals corresponding with ERS-1/2 orbit characteristics. It is shown that differential delays can amount up to more than 1 cm for height intervals of 500 meters or more. For a 2 km height interval and an interferometric baseline of 80 m such delays result in an height error of 180 m. It is not possible to find a generally valid correction scheme for these delays using surface meteorological measurements. Only in situ vertical profile measurements such as radiosondes can be applied to correct for these errors. To obtain a first order indication of the extent of these effects on the accuracy of products derived from radar interferometry, the rms of the delay is determined as a function of height. An empirical expression for the rms is presented.

INTRODUCTION

Repeat-pass spaceborne radar interferometry has been applied successfully for topographic mapping and surface deformation monitoring. Reported accuracies are in the cm or even mm range for surface displacements (Bamler and Hartl 1998) and in the order of 5-15 m for topographic heights, dependent on the effective length of the interferometric baseline (Villasenor and Zebker 1992). Such estimates are based on the accuracy of the phase observation for a single resolution cell, and derived from the estimated or calculated coherence and system theoretical considerations. However, as the main quantity to be derived from radar interferometry is the phase difference between two arbitrary resolution cells, their covariance needs to be accounted for as well. The correlation between two cells is influenced by, e.g., residual orbit error effects or atmospheric delay effects, both resulting in phase gradients in the interferogram.

The influence of the atmosphere on SAR interferograms can be characterized by two driving mechanisms (Tarayre and Massonnet 1996). First, variations of refractivity along a horizontal line at an arbitrary height, during one or both SAR acquisitions, will result in spatial signal delay variations in the interferogram, observed as gradients in the interferometric phase. These effects have been well studied, see, e.g., Goldstein (1995), Massonnet and Feigl (1995), Zebker, Rosen, and Hensley (1997), Hanssen, Weckwerth, Zebker, and Klees (1999), and influence both flat and uneven terrain. Maximum spatial variations of 15 cm over a horizontal distance of 10 km have been reported by Hanssen (1998).

The second mechanism comprises variation of the refractivity along the vertical. Assuming an infinite number of thin atmospheric layers, each with constant refractivity, there will be no horizontal delay differences over flat terrain, even for different refractivity profiles during both SAR acquisitions. This is due to the fact that SAR interferograms are not sensitive to image-wide phase biases. However, for hilly or mountainous terrain a difference in the vertical refractivity profile during both acquisitions will affect the phase difference between two arbitrary resolution cells with different topographic height, see Fig 1, and may cause an erroneous interpretation. This effect has been recognized during deformation studies of mount Etna by, e.g., Tarayre and Massonnet (1996),Massonnet and Feigl (1998),Delacourt, Briole, and Achache (1998), and Ferretti, Prati, and Rocca (1999).

Since the resulting phase error in the phase difference between two resolution cells has zero-mean (expectation value), and the observed phase gradients are often a combination of topographic residuals, deformation, horizontal atmospheric heterogeneity and differential vertical stratification, it has not yet been possible to obtain reasonable error estimates using interferometric data only. A simple and effective means to address this uncertainty is to study vertical radiosonde profiles, and analyze the statistics of the delay variation for every possible height interval. We analyzed 1460 radiosondes acquired 4 times daily during from 1 Jan. 1998 to 31 Dec. 1998, located in a moderate sea-climate at latitude 52.10° and longitude 5.18°. Although climate conditions vary considerably over the globe, it will be

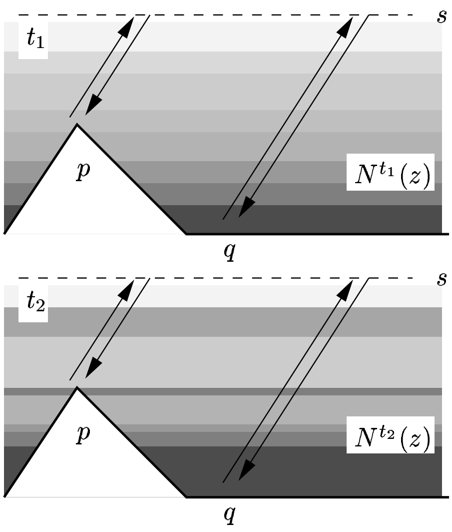

Figure 1: Differential tropospheric delay between point p at the top and point q at the foot of the mountain due to their height difference and a different vertical refractivity profile $N^{t_i}(z)$ during the two SAR acquisitions at t_1 and t_2.

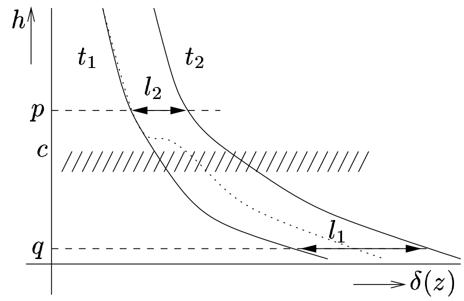

Figure 2: The cumulative delay curves for t_1 and t_2 indicating the height of points p and q, see Fig 1. The effect of the delay on the interferometric phase difference between p and q is determined only by the difference $l_2 - l_1$. The presence of, e.g., an extra cloud layer c at t_1, indicated by the dashed area, will cause a shift of the cumulative delay profile below the cloud, indicated by the dotted line.

shown that it is not the absolute refractivity value, but the dispersion of the refractivity for a fixed height that determines the amount of error for interferometry. As discussed in the sequel, the main factor influencing the variation of refractivity is water vapor. Therefore, we assume that the results reported here can be regarded as representative for a large part of Earth, excluding polar regions, where the atmosphere has a very stable stratification and the cold air cannot hold much water vapor.

THEORY

The geometric and delay terms for points p and q, see Fig. 1, at different topographic heights, during acquisitions t_1 and t_2, assuming zero-baseline, identical incidence angles θ, and no horizontal variation in refractivity N, can be written as:

$$\begin{aligned} \psi_p^{t_i} &= \frac{4\pi}{\lambda\cos\theta}(z_{ps} + \delta_{ps}^{t_i}), \\ \psi_q^{t_i} &= \frac{4\pi}{\lambda\cos\theta}(z_{ps} + z_{qp} + \delta_{ps}^{t_i} + \delta_{qp}^{t_i}), \end{aligned} \quad (1)$$

where z_{ps} is the geometric distance between point p and satellite s, projected to the vertical. The vertical delay between p and s at t_i is indicated by $\delta_{ps}^{t_i}$.

The interferogram phases at points p and q are

$$\begin{aligned} \varphi_p &= \psi_p^{t_1} - \psi_p^{t_2} = \frac{4\pi}{\lambda\cos\theta}(\delta_{ps}^{t_1} - \delta_{ps}^{t_2}), \quad \text{and} \\ \varphi_q &= \psi_q^{t_1} - \psi_q^{t_2} = \frac{4\pi}{\lambda\cos\theta}(\delta_{ps}^{t_1} + \delta_{qp}^{t_1} - \delta_{ps}^{t_2} - \delta_{qp}^{t_2}) \end{aligned} \quad (2)$$

and the contribution of the tropospheric delay on the interferometric phase difference between point p and q is

$$\varphi_{pq} = \varphi_p - \varphi_q = \frac{4\pi}{\lambda\cos\theta}(\delta_{qp}^{t_1} - \delta_{qp}^{t_2}). \quad (3)$$

From (3), we conclude that whenever $\delta_{qp}^{t_1} \neq \delta_{qp}^{t_2}$, there will be a contribution of the tropospheric vertical stratification in the interferogram. Note that this effect will only affect points in the interferogram which have a different topographic height. The total integrated effect is proportional to the height difference.

The delay $\delta_{qp}^{t_i}$ is related to the refractivity profile by

$$\delta_{pq}^{t_i} = 10^{-6}\int_q^p N^{t_i}(z)dz. \quad (4)$$

Hence, we write the interferometric phase difference in (3) as

$$\varphi_{pq} = \frac{4\pi}{\lambda\cos\theta}10^{-6}\int_q^p (N^{t_1}(z) - N^{t_2}(z))dz. \quad (5)$$

The difference in the vertical refractivity profile, eq. (5), can cause a significant contribution in the observed phase difference between point p and q, see the results presented below.

From eq. (4) it is clear that the dimensionless refractivity, integrated over unit distance, yields the fractional delay in parts per million (ppm). For example, $N = 300$ corresponds with a fractional delay of 0.3 mm/m. Therefore, we can regard integrated refractivity values as cumulative delay. The two curves sketched in Fig. 2 indicate the cumulative delay at acquisitions t_1 and t_2. The delay differences between points p and q are $(\delta_q^{t_1} - \delta_p^{t_1})$ and $(\delta_q^{t_2} - \delta_p^{t_2})$ respectively.

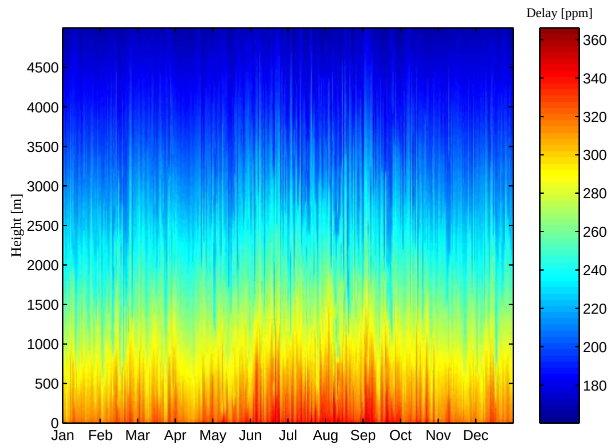

Figure 3: Refractivity or fractional delay in ppm for 365 radiosonde acquired at noon in 1998.

In the interferogram the phase difference, expressed in delay D_{pq}, will be

$$\begin{aligned} D_{pq} &= (\delta_q^{t_1} - \delta_p^{t_1}) - (\delta_q^{t_2} - \delta_p^{t_2}) \\ &= (\delta_q^{t_1} - \delta_q^{t_2}) - (\delta_p^{t_1} - \delta_p^{t_2}) \\ &= (l_2 - l_1). \end{aligned} \quad (6)$$

Hence, to be able to correct for the effect of vertical stratification, it is necessary to know the difference in cumulative delay between t_1 and t_2, both at point p and point q.

RESULTS AND DISCUSSION

Daily radiosonde launches at De Bilt, the Netherlands, measured height, pressure, temperature, and relative humidity during their ascent. Using the Clausius-Clapeyron equation the saturation water vapor pressure corresponding to the temperature level is determined, which enables the conversion from relative humidity to partial water vapor pressure (Stull 1995). Using equations presented in (Smith and Weintraub 1953) we calculated the refractivity for every height level. For all sondes acquired at noon during 1998, the results are shown in Fig 3. Refractivity values are presented as fractional delay in ppm. Apart from seasonal variation it can be observed that the profiles show a ragged behavior, indicating significant variation in refractivity, especially at lower altitudes. Fig. 4 shows the RMS of this variation as a function of height. After interpolation of the refractivity measurements to a regular height interval we can perform the integration in eq. (5) for every combination of two profiles during the year. Simulating different ERS-1/2 repeat intervals, statistics of the contribution of atmospheric delay due to vertical stratification can be obtained for all height intervals and time intervals. Fig. 5 shows

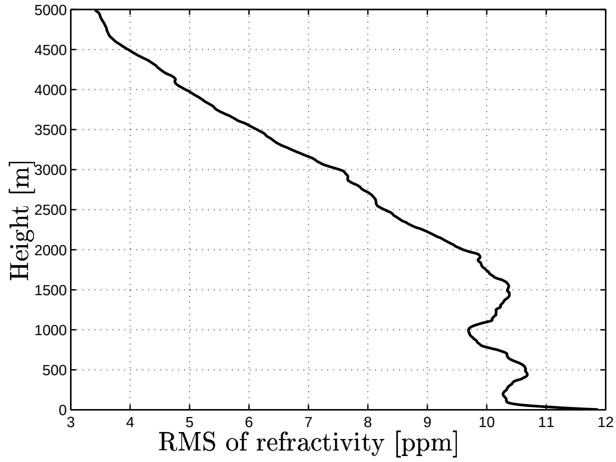

Figure 4: RMS of the refractivity for 365 sondes acquired in De Bilt at noon during 1998, expressed in fractional delay.

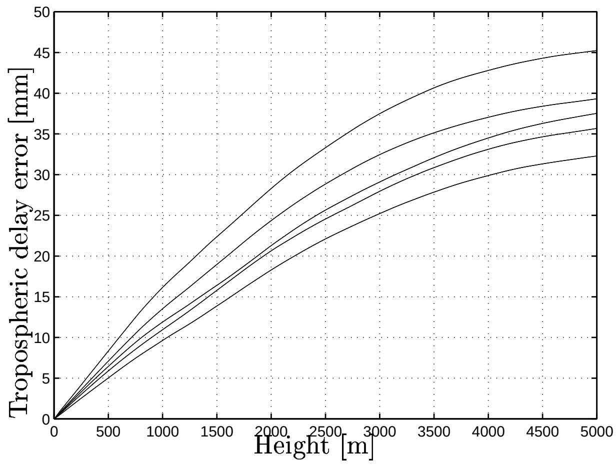

Figure 5: Standard deviation of the observed zenith delay due to vertical stratification as a function of the height interval for the 12 h sonde. The lines represent the 1σ values for 1, 3, 35, 70, and 140 days, counted from the lower to the upper curve.

the standard deviation of the zenith delay difference between a point with zero altitude and points with altitudes up to 5 km. The size of the standard deviation increases for longer time intervals, indicating that 1-day intervals have better characteristics than intervals which exhibit seasonal changes.

With an accuracy better than 2–3 mm we find the following empirical model for the standard deviation of the interferometric phase due to differential tropospheric stratification:

$$\sigma_\varphi = \frac{4\pi}{\lambda\cos\theta}(33.7 + 0.08\Delta t)10^{-3}\sin\frac{h\pi}{2h_s}, \quad \text{for} \quad 1 \leq \Delta t \leq 182, \quad \text{and} \quad 0 \leq h \leq h_s \tag{7}$$

where Δt is the time interval in days, h represents height in m, and $h_s = 5000$ m is a scale height. Above this height the variability of the refractivity is considered negligible. For the accuracy of a height difference one can simply use

$$\sigma_\varphi(\Delta h) = \sigma_\varphi(h_2) - \sigma_\varphi(h_1) \tag{8}$$

Assuming a Gaussian distribution, these results imply that approximately 33% of the interferometric combinations with a specific time interval exhibit effects more severe than expressed in eq. (7). For example, for a time interval of 175 days and a height interval of 2 km, 33% of the interferograms will have more than one phase cycle error due to vertical tropospheric stratification assuming ERS conditions. For a 100 m perpendicular baseline, this translates to a height error of 100 m or worse. For a 1-day interval and 2 km height interval it yields a height error of 76 m or worse.

CORRECTION POSSIBILITIES

Since the size of the error described in the previous section is considerable, it is necessary to investigate methodologies to correct for these errors. Three categories of possibilities are investigated: vertical profile measurements, integrated refractivity measurements, and surface measurements in connection with a model. It is obvious that vertical refractivity profiles, acquired at the interferogram location during the SAR acquisitions are the best option. In this case it is possible to insert the refractivity values in eq. (5) and determine the interferometric phase difference for every pair of pixels. The second option, integrated refractivity or delay measurements can result from, e.g., GPS observations. The problem with this class of observations is that, in order to determine the integrated quantities in eq. (4) and (5), receivers are necessary both in point p and in point q. Therefore, to determine the interferometric phase component due to vertical stratification it is necessary to have such a device at every elevation level, which is highly impractical. The third option, surface observations, has been studied first by Delacourt, Briole, and Achache (1998) on interferometric data of Mt. Etna and assumes that surface observations of pressure, temperature, and relative humidity can be used to approximate the vertical refractivity profile.

The radiosonde data used in this study are ideal to test the hypothesis of retrieving a vertical refractivity profile from surface measurements since an unambiguous comparison between the error signal and the corrected signal can be performed. Therefore, there is no risk of involving other parameters such as errors in the topographic model or due to surface deformation in the results. We tested the Saastamoinen-Baby model, which decomposes the total delay in a hydrostatic component and a wet component (Saastamoinen 1972; Baby, Golé, and Lavergnat 1988). The hydrostatic component approximates the delay component based on surface pressure assuming an exponential decrease with height. The derivative of this delay versus height yields the hydrostatic component of the refractivity. It is well known that this model is very accurate. The wet component of the model is much harder to determine, since there is much more variability with height. Baby, Golé, and Lavergnat (1988) applied the coarse assumption that the relative humidity at the surface remains constant over a certain height interval, above which it reduces to zero. An expression for the wet component of the delay, using two climate-dependent constants, surface relative humidity and temperature linearly decreasing with height can be found in Baby, Golé, and Lavergnat (1988). Differentiating this refractivity-height curve yields the wet component of the refractivity.

Summing the hydrostatic and the wet component of the refractivity yields the 'model'-refractivity $N_m^{t_i}(z)$, which can be compared with the 'true'' refractivity $N^{t_i}(z)$ as obtained from the radiosonde. To study the feasibility to use this model to correct the atmospheric signal due to vertical stratification in the interferogram, we simulated the interferometric error signal by subtracting the refractivity profiles during two days, both for the true as well as the model refractivity, and subtracting both results:

$$N_{res}^{\Delta t}(z) = (N^{t_i}(z) - N^{t_i+\Delta t}(z)) - (N_m^{t_i}(z) - N_m^{t_i+\Delta t}(z)), \tag{9}$$

which yields the residual refractivity $N_{res}^{\Delta t}(z)$ for that simulated interferogram. Comparison of the cumulative sum of $N_{res}^{\Delta t}(z)$ and $N^{\Delta t}(z)$, indicated by $\delta_{pq,res}^{\Delta t}(z)$ and $\delta_{pq}^{\Delta t}(z)$, indicates whether the error signal due to vertical stratification is reduced or not. If for every height level

$$|\delta_{pq,res}^{\Delta t}(z)| \leq |\delta_{pq}^{\Delta t}(z)|, \tag{10}$$

the correction improved the results. Applying this scheme to all interferometric combinations with $\Delta t = [1, 3, 35, 70]$ yielded, with increasing height, 0–50% significant improvement (correction of errors by 4 mm or better). In 0–35% of all simulations, the error signal increased after correction using the surface observations.

The unpredictable behavior of the wet component of the refractivity is the main reason for these disappointing results. In many cases, surface humidity measures do not reflect the humidity of the total profile. An obvious example of such a situation is fog, where surface relative humidity is 100%, which does not necessarily imply high humidities at higher levels. From the analyzed data, it appears that such misinterpretations are common. The fact that interferometric data reflect a linear combination of two different profiles makes it nearly impossible to model the behavior of vertical stratification based on surface observations.

CONCLUSIONS

The accuracy of DEM's obtained from repeat-pass radar interferometry is significantly influenced by atmospheric signal due to turbulent mixing and vertical stratification. An empirical relation between the size of the error due to the latter component has been derived for different height intervals and temporal baselines, using a set of radiosonde observations acquired during 1998. The analyzed data set is restricted to one location on earth, which limits conclusions on global scales. However, since not the total refractivity but the variability of refractivity influence the signal in radar interferograms, it is expected that the results obtained here give a correct order of magnitude of the error.

Correction of the error due to vertical stratification is only possible using vertical profile measurements. Surface observations combined with an tropospheric model are in general unreliable, whereas integrated refractivity observations such as obtained using GPS are insufficient for correction.

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