INTERFEROMETRIC RADAR METEOROLOGY: RESOLVING THE ACQUISITION AMBIGUITY

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ABSTRACT

One of the potential applications of repeat-pass radar interferometry (InSAR) is the monitoring of the lateral atmospheric water vapor distribution. However, the superposition of two atmospheric signals in an interferogram prevents the unambiguous interpretation of the water vapor distribution at a single instant in time. In this paper we present a number of strategies to resolve this acquisition ambiguity, either based on a single master stack or a cascade stack. The cascade based approach has the advantage that large baselines (temporal as well as spatial) can be prevented, reducing decorrelation. For this reason, the cascade stack approach has been applied to a set of ERS 1/2 acquisitions of the Las Vegas Area. The results are satisfying and confirm that radar interferometry can be used for meteorological studies.

INTRODUCTION

One of the potential applications of repeat-pass radar interferometry (InSAR) is the monitoring of the lateral atmospheric water vapor distribution [1]. Main advantages of this application for meteorology are (1) the unprecedented fine resolution (up to 20x20 m), (2) the full vertical column observations (independent of cloud cover) and (3) the very high accuracy of the observations. The main limitation for operational meteorology is currently the long revisit interval per site (in the order of weeks) and often the infrequent acquisition strategy.

One of the key aspects of the technique is its double-difference character. Spatial differences (between resolution cells) result in relative integrated water vapor observations, for absolute values a calibration using a GPS or a radiometer observation should be performed. Secondly, the interferometric temporal differencing (between acquisitions) cause the observed atmospheric phase screen (APS) to be ambiguous. In other words, the interferometric products are a superposition of the atmospheric phase screens at two instances in time.

Two approaches to resolve the acquisition ambiguity are presented: the single master stack and the cascade stack [2]. For both approaches we present the theory, including a derivation of the precision. Some results for a cascade stack of the Las Vegas area are discussed.

THEORY

To estimate the APS per acquisition, the interferograms used are assumed to describe the atmospheric signal only. However, topography, deformation and orbit errors will contribute to the interferometric phase as well. Therefore, the deformation in the area should be minimal, masked or removed after estimation beforehand. Possible orbit errors and hydrostatic gradients often result in a linear trend in the interferogram, which can also be estimated and removed. Finally, the topographic phase can be removed using an external DEM. However, inaccuracies in the DEM will result in a residual phase. Note that this DEM error is dependent on the perpendicular baseline. How the effect of this residual phase can be minimized differs per approach and is therefore addressed separately.

All interferograms have to be co-registered to the same grid of resolution cells. Moreover, for an unambiguous estimation of the APS's the interferograms need to be unwrapped. Obviously, unwrapping errors will influence the final results. All APS estimations are defined on a pixel-by-pixel bases, so all pixels are assumed to be uncorrelated. This results in the advantage that almost all estimation strategies described below consist of simple linear operations, performed on the entire interferograms. Therefore, the estimation processes are very efficient.

Single Master Stack

When $N+1$ radar acquisitions S_i are available, N interferograms I_i can be formed using a single master S_0. Here, the acquisitions S_i are assumed to contain atmospheric delay phase components only. Hence, the surface scattering phases are assumed zero, because they are largely differenced out in the interferometric process anyway. The master should be chosen such that the total sum of perpendicular baselines and temporal baselines is as small as possible. This way, noise due to geometric and temporal decorrelation (remaining scattering phases) is minimized. The relation between the acquisitions and the interferograms for each pixel can be written as (see also Fig. 1A)

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} 1 & -1 & & \\ 1 & & \ddots & \\ \vdots & & & \ddots & \\ 1 & & & & -1 \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ \vdots \\ S_N \end{bmatrix}. \tag{1}$$

Under the assumption that the expectation value of the individual acquisitions S_i is zero, they have equal variance σ^2 and are mutually uncorrelated, the expectation value and dispersion of the interferograms are

$$E\{\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}\} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \text{and} \quad D\{\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}\} = \sigma^2 \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & & 1 \\ \vdots & & \ddots & \vdots \\ 1 & 1 & \cdots & 2 \end{bmatrix}. \tag{2}$$

The strong correlation is obviously due to the use of the same master for each interferogram. This system can also be regarded as a Gauss-Markoff model for the inverse problem — retrieving the APS per acquisition under the assumption that there is no deformation and that the topography is known with sufficient accuracy. Obviously, the system contains a rank deficiency of 1. Hence, there is no unique solution. Two strategies to determine a solution are proposed.

The *first strategy* is simply averaging of the interferograms, obtaining

$$B = \frac{1}{N}\sum_{i=1}^{N} I_i = S_0 - \frac{1}{N}\sum_{i=1}^{N} S_i = S_0 + e, \tag{3}$$

where $E\{e\} = 0$ and $D\{e\} = \frac{\sigma^2}{N}$. Therefore, the estimate for the master acquisition $\hat{S}_0$ equals B and $D\{\hat{S}_0\} = \sigma^2_{\hat{S}_0} = \frac{\sigma^2}{N}$. Alternatively, if the interferograms can be used to estimate the variability of the atmosphere, the averaging could be weighted by the variance of the interferograms, obtaining a more complicated quality description.

After estimation of the master APS, the APS's for the slave acquisitions can be estimated by

$$\hat{S}_i = \hat{S}_0 - I_i \quad \text{with} \quad D\{\hat{S}_i\} = \frac{\sigma^2}{N}. \tag{4}$$

Note that due to the optimal choice of the master acquisition in relation to the slave acquisitions, that is, the sum of perpendicular baselines is minimal, the residual phase due to the DEM error in $\hat{S}_0$ is minimized. However, the DEM error will influence $\hat{S}_i$ and strictly this effect should be added to the precision description above.

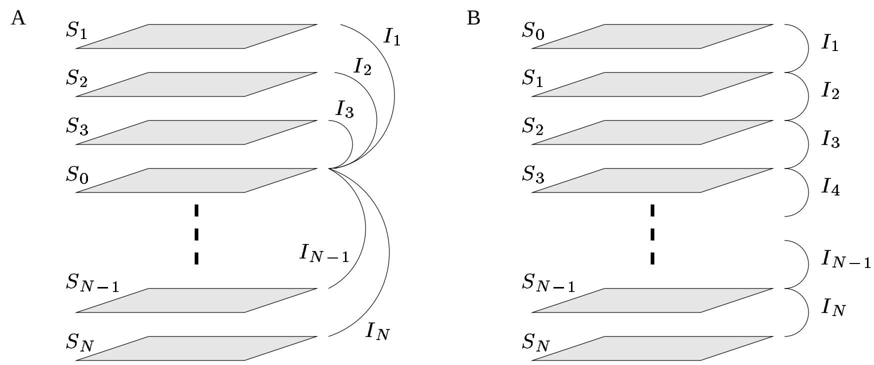

Figure 1: Two stacking approaches. A) Single master stack, where every interferogram is based on the same master. B) Cascade stack, where all acquisitions are sequentially linked.

A *second strategy* to determine a unique solution for the rank deficient problem is based on the Singular Value Decomposition (SVD), which uses a pseudo-inverse to estimate a minimal norm solution. The problem can be formulated as [3]

$$\begin{bmatrix} A^T A & G \\ G^T & 0 \end{bmatrix} \begin{bmatrix} S \\ 0 \end{bmatrix} = \begin{bmatrix} A^T I \\ g \end{bmatrix}, \tag{5}$$

where A is the normal matrix of (1), S are the unknown APS's per acquisition, I are the interferograms and G and g formulate a constraint. In this case, G is a vector of ones and g is zero. The first matrix on the left hand site is now invertible, enabling the determination of the minimum norm solution of S. It can be shown that the estimates for the APS's, including $\hat{S}_0$, read

$$\hat{S}_i = S_i - \frac{1}{N-1}\sum_{i=0}^{N} S_i = S_i + e \quad \text{with} \quad D\{\hat{S}_i\} = \frac{\sigma^2}{N+1}. \tag{6}$$

The SVD-method gives a slightly more accurate estimate of the atmosphere per acquisition compared to the averaging strategy. Again, the effect of the DEM error should be taken into account.

The single master stack enables the use of a large set of interferograms to estimate the APS per acquisitions. However, the temporal and perpendicular baselines can become rather large, resulting in severe noise in the interferograms due to decorrelation. As a result, for atmospheric signal retrieval the usable set of interferograms can become quite limited.

Cascade Stack

The danger of decorrelation described for the single master stack can be partly circumvented using a cascade of interferograms (see Fig. 1B and Fig. 2), where every image (except the first and the last one in the stack) is used two times; once as master and once at slave, see also Fig. 2. The cascade can be formed minimizing the perpendicular and temporal baselines between the acquisitions. Depending on the characteristics of the area, more weight can be given to either the perpendicular or the temporal baseline. This problem resembles the well-known traveling salesman problem, finding the shortest route between points in a two-dimensional space. A simulated annealing algorithm can be used to select an optimal cascade.

The system of observation equations for the cascade stack is

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & \ddots & \ddots & \\ & & & 1 & -1 \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ \vdots \\ S_N \end{bmatrix} \quad \text{with} \quad D\{\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}\} = \sigma^2 \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & \ddots & 0 \\ 0 & \ddots & \ddots & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}. \tag{7}$$

Again, this system can be regarded as a Gauss-Markoff model, with a rank deficiency of 1. This rank deficiency can be circumvented by inserting a pseudo-observable S_k^* in the system. After inversion of the normal matrix, this leads to the solvable system

$$\begin{bmatrix} \hat{S}_0 \\ \vdots \\ \hat{S}_{k-1} \\ \hat{S}_k \\ \hat{S}_{k+1} \\ \vdots \\ \hat{S}_N \end{bmatrix} = \begin{bmatrix} 1 & \cdots & 1 & 1 & & & \\ & \ddots & 1 & 1 & & & \\ & & 1 & 1 & & & \\ & & & 1 & & & \\ & & & 1 & -1 & & \\ & & & 1 & -1 & \ddots & \\ & & & 1 & -1 & \cdots & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_{k-1} \\ S_k^* \\ I_k \\ \vdots \\ I_N \end{bmatrix}. \tag{8}$$

Choosing $S_k^* = 0$ and with $I_i = S_i - S_{i+1}$, the estimates read $\hat{S}_i = S_i - S_k$. Hence, all estimates are biased with a value equal to the atmospheric signal of the reference APS S_k. Obviously, this bias will be smallest by selecting the APS with minimal phase variance, which can be determined by comparing the phase variances of the (neighboring) interferograms. This minimal variance of the interferograms can also be used as an upper bound for the construction of the variance-covariance matrix of the estimated APS's, hence $D\{\hat{S}_i\} = \sigma^2_{I_k}$.

Recalling the averaging approach for the single master stack, a *second strategy* to estimate the APS per acquisition can be deducted. Averaging of the estimated APS's $\hat{S}_i$ obtained above (omitting $\hat{S}_k$) leads to

$$\frac{1}{N}\sum_{0}^{N\backslash k} \hat{S}_i = \frac{1}{N}\sum_{0}^{N\backslash k} S_i - S_k. \tag{9}$$

Because the expectation value of the first part on the right hand side is zero, this average is an estimate for the reference APS. Inserting this estimate as a new pseudo-observable in the functional model, this leads to the new estimates

$$\hat{S}_i = S_i - \frac{1}{N}\sum_{0}^{N\backslash k} S_i. \tag{10}$$

Hence, the bias is now equal to the average of the APS's, omitting the reference APS. Therefore, the bias is smallest if the APS with largest phase variance is selected as reference. The precision of the estimates is $D\{\hat{S}_i\} = \frac{\sigma^2}{N}$. The effect of DEM errors can be minimized by selecting the reference acquisition in the middle of the perpendicular baseline range. This way, the estimation of the reference APS is minimally affected and the baseline length from the reference acquisition to the individual acquisitions is limited as well. The use of a closed cascade is preferable, because in that case the reference acquisition is always in the middle. Besides, the use of linear combinations of interferograms in a network structure might improve the estimates. Again, the effect of the DEM error should be added to the quality description.

Obviously, a weighted averaging could be performed as well. Moreover, only a subset of the estimated APS's can be used for the averaging, for instance based on the variance of the interferograms involved. However, it is difficult to give general recommendations in this perspective, because they are largely dependent on the data set used.

Finally, a *third strategy* to estimate the APS's is the application of the Singular Value Decomposition. The model and the quality description are the same as was described for the single master case. To reduce the computational burden, an iterative scheme can be applied to converge toward the SVD solution without actually computing the time-consuming SVD. The scheme is based on the averaging approach described above. This time, each acquisition is taken as reference sequentially to estimate an improved pseudo-observable for this acquisition. After a number of iterations, the solutions converge to the SVD solution. Now, only basic linear combinations of matrices are used, obtaining a very efficient algorithm.

RESULTS

The cascade approach has been applied using ERS 1/2 data of the Las Vegas (USA) area. Because of the large perpendicular baselines between the acquisitions in the dataset, the single master approach does not give satisfying results for contiguous atmospheric signal retrieval. The area is known for a complex interaction between the atmosphere and the local topography [4]. Therefore, the area is of great interest for meteorologists.

The cascade was formed using a simulated annealing algorithm. Because the area around Las Vegas is supposed to be quite arid, minimal temporal correlation was expected. Therefore, more weight was given to the minimization of the perpendicular baselines. Nevertheless, some interferograms did show strong decorrelation or non-linear artifacts, probably due to orbit inaccuracies. Moreover, the acquisitions obtained after June 2001 are excluded because of the large Doppler shifts. Hence, not all acquisitions are included and the cascade is shorter than intended. The resulting cascade of 33 interferograms is shown in Fig. 2. Some parts of the remaining interferograms showed strong decorrelation (and resulting phase unwrapping errors) in certain parts, most likely due to vegetation and/or snow cover. These areas have been masked (black areas in Fig. 3).

All three cascade strategies described have been applied to the stack. In general, the results obtained show large resemblance. The estimates by the averaging and SVD strategy are very similar and somewhat smoother than the ones obtained by the minimum variance approach. This might be explained by remaining co-registration errors, which are partly averaged out in the averaging and SVD approach. More test cases need to be performed to obtain a better understanding of the characteristics of each approach.

A number of estimated APS's have been selected for meteorological interpretation (see Fig. 3). Fig. 3A shows the passage of a weak cold front over the region. In Fig. 3B mountain induced gravity waves are visible. Fig. 3C shows a topography related signal due to differences in the vertical stratification of refractivity, whereas Fig. 3D reflects a strong convergence line. The general conclusion derived from these images, after comparison with radio soundings, is that InSAR derived water vapor maps per acquisition enable the identification of convergence and vertical motion patterns, irrespective of visible clouds. Moreover, InSAR is able to detect the development of weather systems in an early stage, e.g., the formation of thunderstorms, which are frequently occurring in the area.

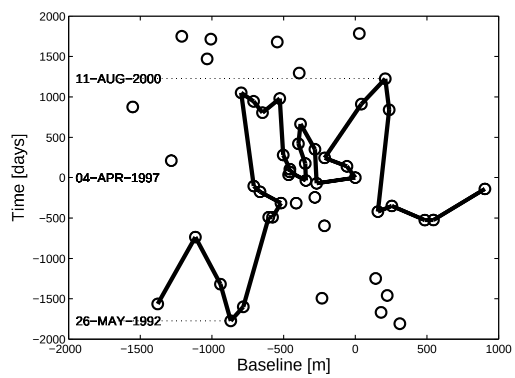

Figure 2: Baseline plot of the cascade used to estimate the APS per acquisition in the Las Vegas area derived by a simulated annealing algorithm. The cascade consists of 33 interferograms.

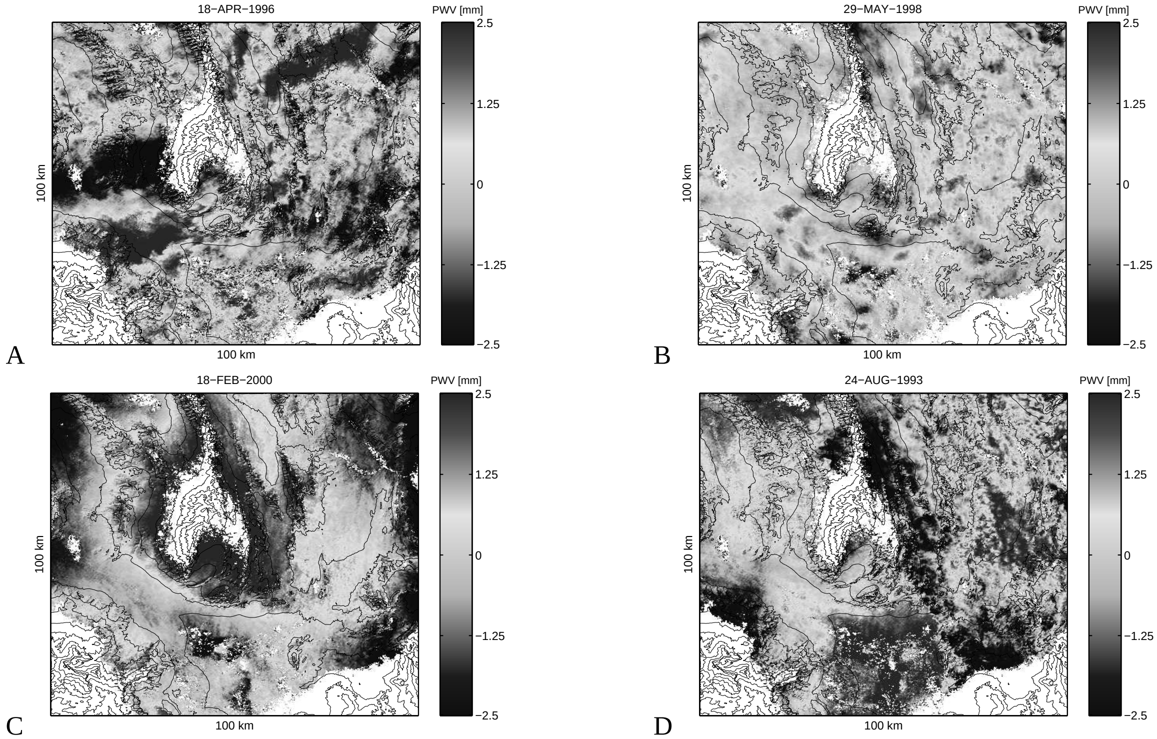

Figure 3: A) Passage of a cold front. B) Mountain induced gravity waves. C) Topography related vertical stratification. D) Strong convergence line. Atmospheric signal is expressed in Precipitable Water Vapor (PWV) [mm]. The black areas are masked because of strong decorrelation due to vegetation and/or snow cover.

CONCLUSIONS

Resolution of the acquisition ambiguity enables the use of radar interferometry for meteorological studies. Two approaches to resolve the acquisition ambiguity have been presented. The advantage of the cascade stack approach is that a large number of acquisitions can be included, with the preservation of small temporal and perpendicular baselines. Because almost all strategies are based on simple mathematical operations (no inversions needed) on a pixel-by-pixel basis, the algorithms are very fast. A test case of the cascade stack approach for the Las Vegas area shows that reasonable estimates for the APS per acquisition can be obtained. Meteorological interpretation is possible and leads to new insights in meso-scale weather phenomena. Future studies will focus on different types of linear combinations of interferograms that could be applied for APS estimation.

ACKNOWLEDGEMENTS

We thank the German Aerospace Center (DLR) and Steven Businger of the University of Hawaii for their help in processing and interpretation of the data.

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