PERSISTENT SCATTERER INTERFEROMETRY: PRECISION, RELIABILITY AND INTEGRATION

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ABSTRACT:

Since the introduction of the persistent scatterer technique (PS-InSAR) the applicability of radar interferometry has increased considerably. As PS-InSAR applications are moving towards low subsidence areas with unfavorable conditions regarding PS density and atmospheric signal, a rigid quality assessment of the estimated parameters is becoming essential. Moreover, a strict quality description enables the integration of PS-InSAR with other geodetic techniques, such as leveling and GPS. Our PS-InSAR processing chain based on geodetic least-squares adjustment techniques combined with integer least-squares estimation to solve the phase ambiguities is described. This approach enables an elegant description of the precision by error propagation and an assessment of the reliability of the estimated parameters. The functional and stochastic framework used are shown, together with the algorithms used. Furthermore, the strategy to integrate PS-InSAR measurements with other geodetic observations is presented.

1 INTRODUCTION

With the introduction of the Persistent Scatterer technique (PS-InSAR) (Ferretti et al., 2000, 2001) the applicability of radar interferometry has increased considerably. Topography and deformation parameters can now be estimated, even in mainly decorrelated areas, using isolated time coherent scatterers. As PS-InSAR applications are therefore moving towards slow subsidence areas with unfavorable conditions regarding PS density and atmospheric signal, a rigid quality assessment of the estimated parameters is becoming essential. Moreover, a strict quality description is required to integrate PS-InSAR with other geodetic techniques, such as leveling and GPS.

2 GEODETIC DATA ADJUSTMENT, TESTING AND QUALITY CONTROL

Random (noise) and systematic errors (biases) in geodetic observations prevent a straightforward transformation of the observations into the parameters of interest. A systematic and rigorous methodology is needed to handle this problem. This methodology consists of three steps: data adjustment, testing and quality control.

Because measurements contain (random) errors, they are adjusted to fit the functional model. The functional model describes the relation between the measurements and the parameters of interest. Together with the stochastic model, which contains the statistical properties of the measurements, it forms the mathematical model. The general form of the mathematical model is denoted in Gauss-Markov form by

$$E\{\underline{y}\} = Ax \quad ; \quad D\{\underline{y}\} = Q_y \tag{1}$$

where the underline denotes the stochasticity of the measurements and

$E\{.\}$	expectation operator
$\underline{y}$	vector of observations
A	design matrix
x	vector of unknowns
$D\{.\}$	dispersion
Q_y	variance matrix

To estimate an optimal solution, the difference between the observations and the model is minimized in a least-squares sense. Linear least-squares estimators that have optimal properties in the sense that they are unbiased and have minimum variance are called Best Linear Unbiased Estimators (BLUE). For normal distributed data, the BLUE estimator is equal to the Maximum Likelihood (ML) estimator (Teunissen, 2000).

Once estimates of the unknown parameters and their variance matrix are obtained, the validity of the mathematical model is tested. That is, the model is tested for errors in the observations $\underline{y}$, in the design matrix A and in the variance matrix Q_y. It is important to realize that it is only possible to test the model when there is redundancy, that is, when the number of observations is larger than the number of un-

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knowns.

Testing is performed by subsequent comparison of two hypotheses: the *null hypothesis* H_0 and the *alternative hypothesis* H_a (Teunissen, 2000). Symbolically the hypotheses can be denoted as

$$H_0: \quad E\{\underline{y}\} = Ax \quad ; \quad D\{\underline{y}\} = Q_y \tag{2}$$
$$H_a: \quad E\{\underline{y}\} = Ax + C\nabla \quad ; \quad D\{\underline{y}\} = Q_y \tag{3}$$

The null hypothesis describes the situation that there are no errors in the model, whereas the alternative hypothesis assumes that there is a certain error. Test statistics are determined and compared to a *critical value* k_α to determine which hypothesis should be rejected.

The impact of (deformation) estimates cannot be evaluated without a quality description. The quality of geodetic products can be parameterized in *precision* and *reliability* terms. Precision describes the variability of the observables and the estimators of the unknown parameters. The precision is quantified in the variance matrix. The reliability describes the sensitivity of the estimators for model errors. Precision and reliability are independent components of the quality description. A high precision of the observables does not imply a reliable estimation of the unknown parameters and vice versa. Precision and reliability together are called *accuracy*.

3 PS-INSAR USING INTEGER LEAST-SQUARES

The most crucial step in PS-InSAR is the correct estimation of the integer valued phase ambiguities. Only when the ambiguities are estimated correctly, the derived deformation parameters will be reliable. Often the ambiguity function (Counselman and Gourevitch, 1981) is used, which performs a discrete search in the solution space. However, because of its optimality property for normal distributed data, here the integer least-squares technique is applied (Teunissen, 1993). The integer least-squares technique is encaptured in the methodology described in Section 2. Hence, an elegant quality description can be obtained, which enables the integration of the PS-InSAR results with other geodetic data.

3.1 Integer least-squares

The basic concept of the integer least-squares technique is to use the knowledge that some parameters, in this case the ambiguities, are integer valued. The problem can be formulated with the extended Gauss-Markov model

$$E\{\underline{y}\} = Aa + Bb, \quad y \in \mathbb{R}, a \in \mathbb{Z}, b \in \mathbb{R}; \quad D\{\underline{y}\} = Q_y, \tag{4}$$

where

A design matrix for integer parameters
B design matrix for real parameters
a vector with integer unknown parameters
b vector with real unknown parameters

The system of equations Eq. (4) is solved in a three step procedure. First, the float solution is computed by neglecting the integer property of the ambiguities. Hence, a standard least-squares adjustment is applied to obtain the estimates $\hat{a}$, $\hat{b}$ and the accompanying variance matrix

$$\begin{bmatrix} Q_{\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}} \end{bmatrix} \tag{5}$$

Then, the ambiguities are resolved in a least-squares sense. To reduce the computation time, the ambiguities are decorrelated using the LAMBDA method (Least-squares AMBiguity Decorrelation Adjustment method)(Teunissen, 1993). The decorrelating transformation reads

$$\hat{z} = Z^T\hat{a}, \qquad Q_{\hat{z}} = Z^T Q_{\hat{a}} Z \tag{6}$$

The integer ambiguities are obtained by solving the minimization problem

$$\min_{z\in\mathbb{Z}} (\hat{z} - z)^T Q_{\hat{z}}^{-1} (\hat{z} - z) \tag{7}$$

Back-transformation gives the fixed ambiguities $\check{a}$.

Once the ambiguities are estimated, the float solution of the parameters of interest $\hat{b}$ is updated using the fixed ambiguities. The fixed solution reads

$$\hat{b}_{|a} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1}(\hat{a} - \check{a}) \doteq \check{b} \tag{8}$$
$$Q_{\check{b}} = Q_{\hat{b}} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} Q_{\hat{a}\hat{b}} + Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} Q_{\check{a}} Q_{\hat{a}}^{-1} Q_{\hat{a}\hat{b}}, \tag{9}$$

where

$$Q_{\check{a}} = \sum_{z\in\mathbb{Z}} (z - a)(z - a)^T P(\check{a} = z) \tag{10}$$

Hence, the variance of the fixed solution is not only dependent on the variance of the float solution, but also on the chance of success in the ambiguity resolution. This chance of success is denoted by the *success rate* $P(\check{a} = a)$. Because of its discrete nature, the fixed ambiguities $\check{a}$ have a probability mass function (PMF) and the fixed solution a multi-modal probability density function (see Figures 1 and 2). Unfortunately, the success rate for the integer least-squares estimator can not be computed in closed form. Therefore, simulations based on the mathematical model are required to determine the success rate.

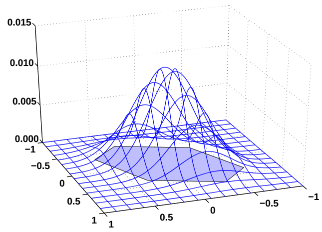

Figure 1. Example of the float ambiguity PDF in 2D

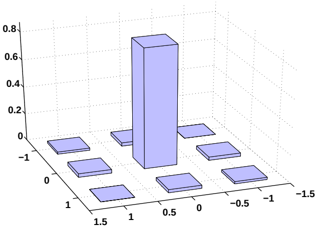

Figure 2. Example of the integer ambiguity PMF in 2D

3.2 Processing chain for PS-InSAR

The processing chain for PS-InSAR is based on the integer least-squares principle. To reduce the effect of atmospheric signal delay and orbit errors, differential phase observations between Persistent Scatterers Candidates (PSC) are used to estimate the ambiguities and the parameters of interest. The mathematical model has the form

$$E\{\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix}\} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} a + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} b; \quad D\{\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix}\} = \begin{bmatrix} Q_{y_1} & 0 \\ 0 & Q_{y_2} \end{bmatrix} \tag{11}$$

where

$\underline{y}_1$	double-difference phase observations
$\underline{y}_2$	pseudo observables
A_1, A_2, B_1, B_2	design matrices
a	unknown ambiguities
b	unknown parameters of interest

The pseudo-observables are required to solve the rank-deficiency of the model. This rank-deficiency is a result of the fact that for each phase observation an ambiguity needs to be estimated. Therefore, no information is left to estimate the parameters of interest. This problem is solved by inserting pseudo-observables y_2 (obvious choice is zero) with high enough variance Q_{y_2} to ensure flexibility.

The ambiguities are resolved using hypothesis tests as described in Section 2 in a recursive scheme. First a linear model is evaluated. If the residues between the model and the unwrapped phase observations are small enough, the assumption is made that the ambiguities are estimated correctly. If not, an alternative deformation model is tested, etc. If necessary, this procedure is repeated for all specified models. In case all deformation models are rejected, the last alternative hypothesis is accepted, which states that the point noise of the arc between the two (PSC) involved is too large. Hence, that at least one of the PSC is not a Persistent Scatterer. Consequently, this arc is removed from the dataset.

The functional model in case only a linear deformation rate is estimated has the form (Kampes and Hanssen, 2004)

$$E\{\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix}\} = \begin{bmatrix} -2\pi I \\ 0 \end{bmatrix} a + \begin{bmatrix} \beta & T \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} H \\ D \end{bmatrix} \tag{12}$$

where

I	identity matrix
β	height-to-phase conversion factor
T	temporal baselines
H	topographic height [m]
D	linear deformation rate [m/year]

This model can easily be adapted with more deformation parameters, e.g. describing a periodic signal. However, for each extra parameter an additional pseudo-observable is required. This will influence the strength of the model and thereby the success rate. However, by testing various deformation models, the chance a good fit will be detected increases, hence increasing the success rate. As a result, more PS are identified. That is, less arcs are rejected due to model noise (deviation of the model from the true but unknown model) instead of point noise (stability of the phase).

After resolving the ambiguities, Eqs. (8 and 9) can be used to estimate the fixed parameters of interest. However, these estimates are be biased by the pseudo-observables and the accompanying variance matrix. Due to the lack of redundancy, these initial values propagate in the float estimates and variance matrices. Therefore, a different approach is followed. The original phase observations are unwrapped using the estimated ambiguities (Kampes, 2005)

$$\underline{y}_1^{\text{unw}} = \underline{y}_1 - A_1 \check{a} \tag{13}$$

leaving the system of equations

$$E\{\underline{y}_1^{\text{unw}}\} = B_1 b; \quad D\{\underline{y}_1^{\text{unw}}\} = Q_{y_1^{\text{unw}}} = Q_{y_1} \tag{14}$$

This implies the assumption that the ambiguities are estimated correctly, hence, that the success rate equals one. To check this, the full network is analyzed for closing errors of the ambiguities using hypotheses tests. If needed, non-fitting arcs are corrected or removed. As an example, Figure 3 shows a network for a small part (2×10 km) of Las

Vegas. Non-fitting arcs (red) are removed. Once there are no more closing errors, the ambiguities are assumed to be unwrapped correctly and the success rate is set to one. Obviously, loose arcs can not be tested and are removed from the data set.

Now the ambiguities are assumed deterministic, the parameters of interest can easily be calculated using a standard least-squares estimator

$$Q_{\check{b}} = (B_1^T Q_{y_1}^{-1} B_1)^{-1} \quad (15)$$

$$\check{b} = Q_{\check{b}} B_1^T Q_{y_1}^{-1} \underline{y}_1^{\text{unw}} \quad (16)$$

With this, estimates for the topography and deformation are obtained, together with variance matrices describing the precision. However, the unwrapped phases (Eq. (13)) can also be further processed, e.g., to filter atmospheric effects and/or to integrate PS-InSAR with other geodetic techniques. This is described in the next section.

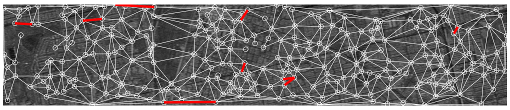

Figure 3. Network of PS for a small area in Las Vegas (2×10 km).

4 DATA INTEGRATION

Integration of PS-InSAR measurements with observations from other (geodetic) techniques is not straightforward. This paragraph describes the main issues to resolve for integration of measurements from different techniques for parameter estimation including a quality description.

4.1 The observations

A precise measurement is not necessarily a reliable measurement. When combining observations from different techniques the importance of the reliability concept increases: what exactly are we measuring? What is the physical shape of the measurement point and what kind of surface deformation does it quantify? Data integration therefore starts with the analysis of the observations themselves. One has to realize that observations from different techniques are often both physically and information-wise not the same. To demonstrate this, PS-InSAR is compared to the leveling technique, which is commonly used for validation purposes.

4.1.1 Physical representation of the observations

Leveling measurements refer to well-defined benchmarks mounted on a (stable) subsurface layer, whereas PS-InSAR measurements are summed-up reflections from a resolution cell containing a dominant scatterer. The physical representation of the PS measurement depends on the location and the type of reflection.

To locate the PS 'measurement point,' is it necessary to quantify the PS location uncertainty in geographical coordinates. The PS location uncertainty depends on accuracy of the reference point height, orbital inaccuracies and acquisition timing errors, estimated to be between 1 and 8 m.

To classify the type of reflection there are several options. First, polarization may be used to distinguish between even and odd bounced scatterers. Such a procedure is usually not possible, as most acquisitions are in single polarization mode. If alternating polarization is available, the same PS have to be identified. A second method is the accurate coregistration of the PS locations and their estimated heights with a geographical database. Then PS are compared with a 3D city model, to determine where the dominant reflection physically stems from, e.g., roof reflection or curb-to-wall double bounces. Additional information is required from the area, such as classified topographic maps in vector and/or raster format and laser altimetry data. To perform this comparison, precision estimates of the PS locations and their estimated (residual) heights are required, as well as the accuracy of the additional maps.

4.1.2 Information contents of the observations

Information-wise, PS-InSAR and leveling observations are not the same, as they differ in spatio-temporal contents. A leveling measurement $\underline{h}_{ij}^t$ is a spatial height difference between points i and j on time t. A PS-InSAR measurement $\underline{d}_i^{mt}$ is a temporal interferometric difference between master time m and slave time t, for a certain point i. The first interpretable PS-InSAR observation is the double-difference $\underline{d}_{ri}^{mt}$, both temporally (between master and slave) and spatially (with respect to a reference point r) (Hanssen, 2004). They can be reformulated in the form of leveling type of observations by setting the spatial deformation in the master image to 0:

$$\underline{d}_{ri}^{mt} = \underline{d}_{ri}^{m} - \underline{d}_{ri}^{t} \quad ; \quad \underline{d}_{ri}^{m} = 0 \quad \rightarrow \quad \underline{d}_{ri}^{mt} = -\underline{d}_{ri}^{t} \quad (17)$$

On the other hand, it is also possible to construct double-differences from the leveling observations:

$$\underline{h}_{ij}^{t_1 t_2} = \underline{h}_{ij}^{t_1} - \underline{h}_{ij}^{t_2} \quad (18)$$

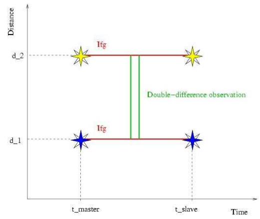

Figure 4. PS-InSAR double-differences.

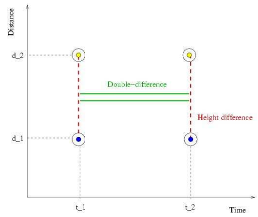

Figure 5. Leveling double-differences.

Figure 4 and 5 show the difference between PS-InSAR and leveling double-differences. Note that a leveling (spatial) single difference can be measured directly, whereas for the PS-InSAR temporal single difference, an interferogram has to be created.

4.2 Geometric and orthometric heights

The type of heights resulting from different measurement techniques, like PS-InSAR and leveling, may also differ. We distinguish between geometric (ellipsoidal) and orthometric height. A leveling instrument is always positioned perpendicular on the gravity field. Therefore heights are measured relative to an equipotential level surface. The equipotential surface of the earth's gravity field that follows the global mean sea level is called the geoid. Heights measured relative to the geoid are called orthometric heights.

GPS, PS-InSAR and orbital heights are heights referring to a mathematical ellipsoidal shape of the earth. The difference between ellipsoid and geoid can be determined from gravity measurements and results in a geoid model, e.g., EGM96. The difference between ellipsoid heights (PS-InSAR) and orthometric heights (leveling) is equal to the geoid height (geoid undulation):

$$N = H - h \tag{19}$$

where

N	geoid height
H	ellipsoidal height (PS-InSAR)
h	orthometric height (leveling)

As the observations are spatial differences, the relative accuracy of two nearby points is of importance. This is generally much higher than the accuracy of the geoid itself (centimeter-decimeter level). To convert PS-InSAR deformation measurements to the leveling height system, they have to be converted from slant-range to the vertical and from ellipsoidal to orthometric displacements.

4.3 Estimation in phases

Because of the different physical properties of the measurement points, benchmark versus reflection, it is not possible to compare both measurements directly. The integrated use of leveling and PS-InSAR measurements arises in the joint estimation of the unknown (deformation) parameters. The question arises where to start the data integration for PS-InSAR? Which PS-InSAR observation is most suitable to merge with observations from other techniques: the raw SAR observations, the interferometric phase, the double-difference or the deformation estimate? It should not matter which starting observations are taken, a long as their variance matrix is propagated correctly.

To answer the question of which PS-InSAR observation to use in the integration, we start from the mathematical model as defined in equation 14:

$$\begin{aligned} E\{\underline{y}_1^{\text{unw}}\} &= B_1 b \\ D\{\underline{y}_1^{\text{unw}}\} &= Q_{\underline{y}_1} = Q_{\text{atmo}} + Q_{\text{defo,res}} + Q_n \end{aligned}$$

Based on this model, we predict the realization of the stochastic deformation vector, which is used as vector of observations in the data integration. Here, the Best Linear Unbiased Prediction (BLUP) theory (Teunissen et al., 2005) is applied based on the Best Linear Unbiased Estimator $\underline{\check{b}}$ and its variance matrix. The BLUP problem can be structured as a partitioned linear model where only $\underline{y}$ is observed and $\underline{z}$ needs to be predicted:

$$\begin{bmatrix} \underline{y}_1^{\text{unw}} \\ \underline{z} \end{bmatrix} = \begin{bmatrix} B_1 \\ B_{\text{defo}} \end{bmatrix} b + \begin{bmatrix} \underline{e} \\ \underline{s}_{\text{defo,res}} \end{bmatrix} \tag{20}$$

where

$\underline{z}$	PS-InSAR deformation vector to be predicted
B_{defo}	design matrix specifying the relation to the deformation parameters
$\underline{e}$	residual vector, consisting of atmospheric signal, residual deformation signal and measurement errors
$\underline{s}_{\text{defo,res}}$	residual deformation signal

The propagation law results in the variance matrix:

$$\begin{bmatrix} Q_{yy} & Q_{yz} \\ Q_{zy} & Q_{zz} \end{bmatrix} = \begin{bmatrix} Q_{y_1}^{\text{unw}} & Q_{\text{defo,res}} \\ Q_{\text{defo,res}} & Q_{\text{defo,res}} \end{bmatrix} \tag{21}$$

The matrix B_{defo} contains zero columns corresponding with the parameters in $\check{\underline{b}}$ that are not related to deformation, e.g., height. This does not cause problems, as this matrix is not involved in any inversion.

Applying the BLUP theory results in a predicted deformation vector and its error variance matrix, which are the PS-InSAR observations and their dispersion in the integrated mathematical model:

$$\check{\underline{z}} = B_{\text{defo}}\check{\underline{b}} + \check{\underline{s}}_{\text{defo,res}} \tag{22}$$

$$P_{\check{z}\check{z}} = Q_{zz} - Q_{zy}Q_{yy}^{-1}Q_{yz} + (B_{\text{defo}} - Q_{zy}Q_{yy}^{-1}B_1) \\ Q_{\check{b}\check{b}}(B_{\text{defo}} - Q_{zy}Q_{yy}^{-1}B_1)^T \tag{23}$$

The main difficulties in predicting the deformation observation vector are:

- the covariance functions of the atmosphere and the residual deformation signal for constructing Q_{atmo} and $Q_{\text{defo,res}}$,
- the dimensions of the adjustment problem.

The covariance function describes the spatial and residual behavior of the deformation signal, which means that either knowledge about the deformation signal is required, or that redundancy and network construction has to be sufficient to estimate the stochastic model parameters. The dimension of the adjustment problem is too large to calculate the solution at once, especially for the matrix inverses to be computed. This means that the adjustment problem has to be split up in (groups of) arcs, and as a result a large number of connection adjustments have to be solved additionally.

4.4 PS-InSAR and leveling integration in the parameter space

This paragraph describes the mathematical model for combining PS-InSAR with leveling measurements for the joint estimation of deformation parameters in the presence of several deformation regimes. This mathematical model is based on estimators and predictors which are optimal in the sense that they are unbiased and have minimum error variance.

The relation between measurements and unknown deformation parameters can be written in the following way:

$$\begin{bmatrix} \underline{y}_{\text{lev}} \\ \underline{y}_{\text{psi}} \end{bmatrix} = \begin{bmatrix} \underline{y}_{\text{lev}} \\ \check{\underline{z}} \end{bmatrix} = Ax + \sum_{d=1}^{D} \underline{s}_d(x,y,t) + \begin{bmatrix} \underline{n}_{\text{lev}} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \underline{n}_{\text{psi}} \end{bmatrix} \tag{24}$$

where

$\underline{y}_{\text{lev}}$	leveling observations
$\underline{y}_{\text{psi}}$	PS-InSAR observation vector, which is equal to the predicted deformation $\check{\underline{z}}$
A	design matrix defining the relation between observations and unknown deformation parameters
x	unknown deformation parameters
$\underline{s}_d(x,y,t)$	signal describing the discrepancy between model and actual deformation for deformation regime d with a certain spatial (x,y) and temporal (t) behavior
$\underline{n}_{\text{lev}}$	measurement error leveling
$\underline{n}_{\text{psi}}$	measurement error PS-InSAR

This can be written as a combined functional and stochastic model:

$$E\{\underline{y}\} = Ax; \quad Q_y = \sum_{d=1}^{D} Q_{s_d(x,y,t)} + \begin{bmatrix} Q_{n_{\text{lev}}} & 0 \\ 0 & Q_{n_{\text{psi}}} \end{bmatrix} \tag{25}$$

where $Q_{n_{\text{psi}}}$ equals $Q_{\check{z}\check{z}}$.

Best Linear Unbiased Estimators (BLUE) for the deformation parameters and their variance matrix can then be calculated. The (residual) signals due to different deformation regimes can be predicted using the BLUP theory, see Eq. (20). However, the application of this theory is not straightforward. The indentified difficulties are:

- the choice of the unknowns and the possible non-linear relation with the observations,
- rank deficiencies,
- different nature of the (double-difference) observations from different techniques,
- determination of the deformation regime covariance functions,

- determination of the PS-InSAR measurement noise.

A non-linear relation between observations and unknowns is commonly solved by linearizing the equations and iteratively solving the adjustment problem.

Rank deficiencies may be inherent to the relation between the observations and the unknowns. For example, heights can never be estimated from height differences, nor from height double-differences. To compute heights from height differences, the height of one reference point serves as an S-basis (Baarda, 1981). However, it does not matter which reference point is chosen, as the adjustment results are intrinsically the same.

Although leveling height double-differences and PS-InSAR double-differences quantify the same deformation double difference, they may need their own S-basis. This occurs for example if the unknowns are PS and benchmark 'heights'. As a PS and a benchmark are never physically the same, two S-bases have to be defined. In this case the leveling and PS-InSAR measurements are only connected stochastically through the spatio-temporal deformation regimes.

Regarding the covariance functions for the deformation regimes, the same as for the PS-InSAR estimation holds: knowledge about the deformation regimes is required and stochastic parameters can only be estimated with a high precision if there is enough redundancy in the mathematical model.

As the measurement precision of a PS cannot be separated from noise due its physical properties (signal against clutter in the surroundings) it is difficult to estimate the precision of a PS observation. However, through Variance Component Estimation (Teunissen, 1988), stochastic model parameters can be estimated. The validity of the PS stochastic model based on Signal-to-Clutter ratio (SCR) has been tested for the five corner reflectors deployed in Delft using the independent leveling technique (Ketelaar et al., 2004). The results show an overestimation of the a-priori SCR phase variance, which indicates that the stochastic model has to be treated carefully.

5 CONCLUSION

The use of integer least-squares in PS-InSAR enables a systematic and rigorous assessment of the precision and reliability of the derived parameters. Standard geodetic data adjustment, testing and quality description methods can be applied. With the quality description obtained, it is possible to integrate PS-InSAR observables with other geodetic data such as leveling and GPS. Because the physical location of the measurement points differs, it is only possible to integrate the techniques in the joint estimation of the deformation parameter.

The most crucial step in PS-InSAR is the correct estimation of the integer valued phase ambiguities. Only when the ambiguities are estimated correctly, the derived deformation parameters will be reliable. Up till now the use of spatial correlation between PS is restricted to single arcs. Future research will focus on incorporation of the spatial correlation between nearby PS to increase the success rate of correct integer estimation.

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