

SPATIO-TEMPORAL PHASE UNWRAPPING USING INTEGER LEAST-SQUARES

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ABSTRACT

State-of-the-art phase unwrapping techniques for interferometric time series do not fully benefit from the spatio-temporal correlations between the phase observations. Incorporation of these correlations in the unwrapping process may increase the success rate of correct unwrapping. Here, the mathematical framework for 3D spatio-temporal phase unwrapping using integer-least squares is presented. The technique is based on the 1D (temporal unwrapping) + 2D (spatial unwrapping) approach, currently applied in most Persistent Scatterer Interferometry (PSI) algorithms. The stochastic structures as well as the functional models are shown.

1 INTRODUCTION

With the growth of the data archives, interferometric radar applications for deformation monitoring shifted from single interferogram to time series analysis. Examples of the latter are the Persistent Scatterer technique (PSI) [1, 2] and the optimal subsets approaches, e.g., [3, 4, 5]. Within the PSI technique the deformation of isolated point scatterers with stable phase behavior is estimated using a single master stack. On the other hand, the optimal subsets approaches use multi-looked interferograms with favorable baseline characteristics, grouped in subsets, to extract the deformation signal. Regardless the technique chosen, correct phase unwrapping remains the crucial step for a reliable analysis. So far, two main strategies for the unwrapping of the time series can be distinguished: 1) $N \cdot 2D$ unwrapping using conventional techniques, e.g., [3, 4], and 2) 1D+2D unwrapping, e.g., [2, 5]. Obviously, unwrapping in multiple steps is suboptimal, as spatial and temporal correlations are not optimally taken into account. Here, the mathematical framework for an integrated 3D phase unwrapping algorithm is presented, based on the 1D+2D approach, which is discussed first.

2 1D+2D UNWRAPPING

Within the 1D+2D approach, the time series of individual resolution cells are first independently unwrapped in time, followed by a spatial unwrapping step. To reduce the effect of atmospheric distortions in the temporal unwrapping step, often the differential phases between two nearby resolution cells are analyzed. These differential phases are denoted as double differences (DD), differences in time and space. The original interferometric phases are regarded as single differences (SD).

2.1 Temporal unwrapping

Various temporal phase unwrapping techniques are proposed, each with their own optimization criterion and stochastic characteristics. Examples are the ambiguity function [6, 1], integer bootstrapping, and integer least-squares [7, 8]. The ambiguity function searches a discrete solution space for maximal temporal coherence, assuming a predefined functional model under the null-hypothesis (e.g. linear deformation). Because no stochastic properties of the phases are considered, the ambiguity function is a deterministic estimator. Using the integer bootstrapping technique, some of the correlation between the observations is taken into account. This implies that the stochastic model, describing the stochastic properties of the observations, needs to be known. An advantage of bootstrapping is that additional deformation parameters can easily be added. However, with the extra parameters the strength of the mathematical model is further reduced. The same holds for the integer least-squares (ILS) technique. However, here the full stochastic model is considered. For Gaussian distributed data, ILS is shown to be equivalent to the Maximum Likelihood estimator [9]. Because of this property and the large flexibility in deformation models that can be considered, the integer least-squares technique is applied here.

The basic concept of the integer least-squares technique is to acknowledge the fact that some parameters, in this case the

ambiguities, are integer valued. The ILS problem can be denoted by the extended Gauss-Markov model as

$$E\{\psi\} = Aa + Bb, \quad D\{\psi\} = Q_\psi, \quad (1)$$

and the least-squares estimations are obtained solving the minimization problem

$$\min_{a,b} \|\psi - Aa - Bb\|_{Q_\psi}^2, \quad \psi \in \mathbb{R}, a \in \mathbb{Z}, b \in \mathbb{R}, \quad (2)$$

where $E\{\cdot\}$ is the expectation operator, $D\{\cdot\}$ the dispersion, ψ the vector of phase observations, A and B are design matrices for the integer and real valued parameter vectors a and b respectively, and Q_ψ is the covariance matrix. The minimization problem Eq. (2) is solved in a three step procedure. First, the float solution is computed by neglecting the integer property of the ambiguities. Hence, a standard least-squares adjustment is applied to obtain the estimates \hat{a} , \hat{b} and the accompanying variance matrix

$$\begin{bmatrix} Q_{\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}} \end{bmatrix}. \quad (3)$$

Then, the ambiguities are resolved in a least-squares sense. To reduce the computation time, the ambiguities are 'decorrelated' using the LAMBDA method (Least-squares AMBiguity Decorrelation Adjustment method)[7, 10]. The decorrelating transformation reads

$$\hat{z} = Z^T \hat{a}, \quad Q_{\hat{z}} = Z^T Q_{\hat{a}} Z. \quad (4)$$

The integer ambiguities are obtained by solving the minimization problem

$$\min_{z \in \mathbb{Z}} (\hat{z} - z)^T Q_{\hat{z}}^{-1} (\hat{z} - z). \quad (5)$$

Back-transformation yields the fixed ambiguities \check{a} .

Once the ambiguities are estimated, the float solution of the parameters of interest \hat{b} is updated using the fixed ambiguities. The fixed solution reads

$$\hat{b}_{|a} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} (\hat{a} - \check{a}) \doteq \check{b}, \quad (6)$$

$$Q_{\check{b}} = Q_{\hat{b}} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} Q_{\hat{a}\hat{b}} + Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} Q_{\hat{a}} Q_{\hat{a}}^{-1} Q_{\hat{a}\hat{b}}, \quad (7)$$

where

$$Q_{\check{a}} = \sum_{z \in \mathbb{Z}} (z - a)(z - a)^T P(\check{a} = z). \quad (8)$$

Hence, the variance of the fixed solution is not only dependent on the variance of the float solution, but also on the chance of success in the ambiguity resolution. This chance of success is denoted by the *success rate* $P(\check{a} = a)$. Because of their discrete nature, the fixed ambiguities \check{a} have a probability mass function (PMF) and the fixed solution a multi-modal probability density function. Unfortunately, the success rate for the integer least-squares estimator can not be computed in closed form. Therefore, simulations based on the mathematical model are required to estimate the success rate.

Stochastic model for PSI

The covariance matrix Q_ψ for the double difference phase observations are derived from the original Single Look Complex (SLC) observations φ by error propagation. The SLC observations are denoted as

$$\overline{\varphi} = \begin{bmatrix} \overline{\varphi}_i \\ \overline{\varphi}_j \end{bmatrix} = [\varphi_i^0 \quad \cdots \quad \varphi_i^N \quad \varphi_j^0 \quad \cdots \quad \varphi_j^N]^T, \quad Q_{\overline{\varphi}} = Q_{\text{coh}} + Q_{\text{atmo}} + Q_{\text{defo}}, \quad (9)$$

where i, j indicate the points, N is the number of SLCs (the master image is denoted by a zero), Q_{defo} is the covariance matrix due to residual (non-modeled) deformation, Q_{atmo} describes the variability of the atmosphere and Q_{coh} represents the noise due to loss of coherence and, e.g., thermal noise. Examples of the structure are shown in Fig. 1A-D. The covariance matrices are derived using a covariance function, where the one for deformation is a function of time and distance and the atmospheric function is only dependent on distance. The power law behavior of atmospheric turbulence [11, 12] is used to construct the function based on the Matérn class of covariance functions [13].

The covariance matrix of the interferograms (SD) is obtained by

$$\overline{\phi} = \begin{bmatrix} \overline{\phi}_i \\ \overline{\phi}_j \end{bmatrix} = \Lambda \overline{\varphi}, \quad Q_{\overline{\phi}} = \Lambda Q_{\overline{\varphi}} \Lambda^T, \quad \Lambda = [I_2 \otimes [e_N \quad -I_N]], \quad (10)$$

Testing on the ambiguities

Testing on the ambiguities is possible on the condition that the constructed double difference phase observations are not re-wrapped. This can be seen as follows. Assume an interferogram in which 3 points are selected (see Fig. 2). Each point has an unwrapped interferometric phase Φ , composed by an unknown ambiguity a and an observed wrapped phase ϕ .

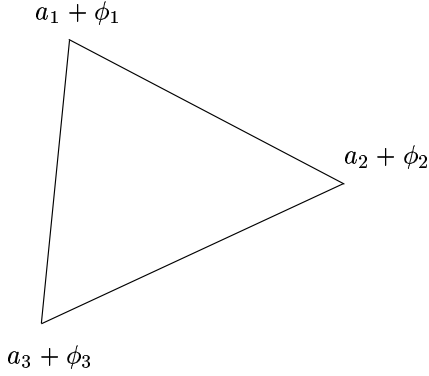


Figure 2: Triangle of interferometric phases, composed by an unknown ambiguity a and an observed wrapped phase ϕ .

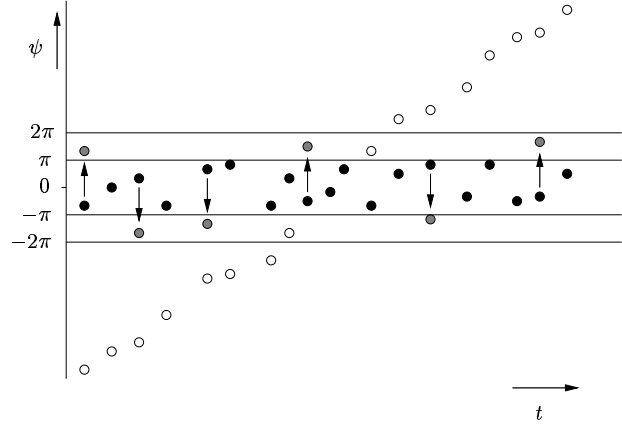


Figure 3: The effect of not re-wrapping double difference phase observations for a single arc.

Constructing the double differences results in

$$\begin{aligned}
 \Psi_{12} &= (a_2 + \phi_2) - (a_1 + \phi_1) = (a_2 - a_1) + (\phi_2 - \phi_1) = \Delta a_{12} + \psi_{12}, \\
 \Psi_{23} &= (a_3 + \phi_3) - (a_2 + \phi_2) = (a_3 - a_2) + (\phi_3 - \phi_2) = \Delta a_{23} + \psi_{23}, \\
 \Psi_{31} &= (a_1 + \phi_1) - (a_3 + \phi_3) = (a_1 - a_3) + (\phi_1 - \phi_3) = \Delta a_{31} + \psi_{31},
 \end{aligned} \tag{13}$$

where Ψ are the unwrapped DD phases. Note that $\psi_{12} \in [-2\pi, +2\pi)$. By definition, these unwrapped phases form a closed loop. As the ψ are not re-wrapped, their sum is also zero. Consequently, the ambiguities Δa form a closed loop as well, which can be used in the testing. Wrapping of the DD phases ψ will result in phase residues, and the closure of the ambiguities no longer holds. Hence, re-wrapping of phases results in a loss of information for this application. This in contradiction to branch-cut unwrapping methods, where the phase residues are the prime source of information. The consequence of not re-wrapping the DD phases is illustrated in Fig. 3 for a single time series.

Testing based on the ambiguities has the advantage that type B unwrapping errors can be detected and corrected, thereby preserving the strength of the network as much as possible. Moreover, the number of deformation parameters per arc does not influence the testing. Hence, the drawbacks of testing based on the parameters are accounted for.

After the testing procedure, the success rate of correct ambiguity resolution is assumed to be one. This results in a variance matrix of the fixed ambiguities Eq. (8) equal to zero, which causes that the last term of Eq. (7) vanishes. Hence, normal statistics and error propagation can be applied, enabling for example the integration of PSI results with other geodetic measurements.

3 3D UNWRAPPING

Although the 1D+2D unwrapping approach may lead to satisfying results, this is not guaranteed. When too many arcs are rejected in the spatial unwrapping step, the redundancy vanishes and the reliability of the solution decreases. An integrated 3D approach circumvents these problems. Instead of temporal unwrapping of single arcs, within the 3D unwrapping algorithm multiple arcs are unwrapped simultaneously, benefiting from the mutual correlations. The closure of the ambiguities in case of not re-wrapped DD is used again in the estimation process. An example is derived for the small network shown in Fig. 4.

Stochastic model

The stochastic model for the 3D situation is derived in a similar manner as for the 1D case. The covariance matrix of the

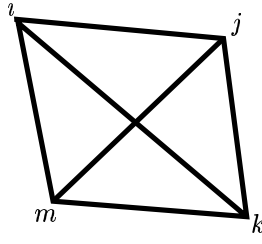


Figure 4: Example of network for 3D phase unwrapping.

interferograms is

$$\bar{\phi} = \begin{bmatrix} \bar{\phi}_i \\ \bar{\phi}_j \\ \bar{\phi}_k \\ \bar{\phi}_m \end{bmatrix} = \Lambda \bar{\varphi}, \quad Q_{\bar{\phi}} = \Lambda Q_{\bar{\varphi}} \Lambda^T, \quad \Lambda = [I_4 \otimes [e_N \quad -I_N]], \quad (14)$$

and the covariance matrix for the double differences becomes

$$\bar{\psi} = \Omega \bar{\phi}, \quad Q_{\bar{\psi}} = \Omega Q_{\bar{\phi}} \Omega^T, \quad \Omega = \begin{bmatrix} \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \otimes I_N \end{bmatrix}. \quad (15)$$

Because of the correlations between the DD phase observations, the result of Eq. (15) is singular. An extra regularization parameter, which results in an extra term on the main diagonal of $Q_{\bar{\psi}}$, solves this problem. Figure 5 shows the structures of the covariance matrices involved.

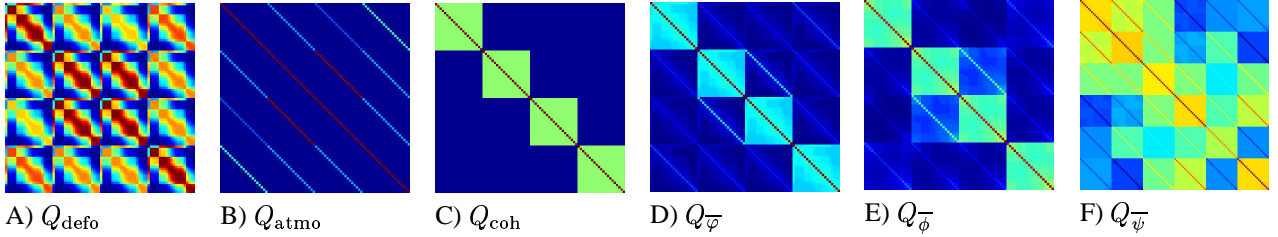


Figure 5: Structures of covariance matrices for a network of multiple arcs. A) Non-modeled deformation Q_{defo} (function of time and distance), B) Atmosphere Q_{atmo} (function of distance), C) Coherence and e.g., thermal noise Q_{coh} , D) SLC phase $Q_{\bar{\varphi}}$, E) interferometric phase $Q_{\bar{\varphi}}$ (single difference), and F) double difference phase $Q_{\bar{\psi}}$.

Functional model

The functional model for the integrated 3D phase unwrapping is

$$E \left\{ \begin{bmatrix} \bar{\psi} \\ \bar{H}^* \\ \bar{D}^* \end{bmatrix} \right\} = \begin{bmatrix} \Omega' \otimes -2\pi I_N \\ 0 \\ 0 \end{bmatrix} \bar{a} + \begin{bmatrix} \Omega' \otimes \bar{\beta} & \Omega' \otimes \bar{\alpha}(\bar{t}) \\ I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{H} \\ \bar{D} \end{bmatrix}. \quad (16)$$

The matrix Ω' is the connection matrix without the first column, which accounts for the required reference point in the network (see Eq. (12) for the analogy with the single arc case).

Practical implementation

The more arcs are added to the network, the larger the 3D unwrapping problem becomes. Obviously, computational restrictions limit the maximum network size. To overcome this problem, a hierarchic unwrapping sequence can be applied. First, a global network is unwrapped with a limited number of points. Once unwrapped, these points are used to constrain

the unwrapping of other points, and so on. An alternative is a region growing algorithm, again using constraints. The functional model with constraints on the ambiguities is

$$E \left\{ \begin{bmatrix} \bar{\psi} \\ \bar{H}^* \\ \bar{D}^* \\ \bar{a}^* \end{bmatrix} \right\} = \begin{bmatrix} \Omega' \otimes -2\pi I_N \\ 0 \\ 0 \\ I \end{bmatrix} \bar{a} + \begin{bmatrix} \Omega' \otimes \bar{\beta} & \Omega' \otimes \bar{\alpha}(\bar{t}) \\ I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{H} \\ \bar{D} \end{bmatrix}. \quad (17)$$

Preliminary tests on simulated data show the effectiveness of this approach. However, computation time of ILS is a limiting factor. Strong improvements in this perspective are obtained by using integer bootstrapping, although part of the correlations are hereby neglected. The next step will be the full implementation of the algorithm and tests on real data.

4 CONCLUSIONS

Three-dimensional phase unwrapping using integer least-squares fully benefits from the mutual correlations between the phase observations in time and space. The technique is based on the 1D+2D unwrapping approach currently used in most PSI algorithms. Innovative here is the testing the spatial network based on the estimated ambiguities. This not only enables the detection of unwrapping errors, but also the isolation and correction of single erroneous ambiguities. The same constraints on the ambiguities are used in the 3D approach. Although tests on simulated data are satisfying, applications on real data have to show the reliability of the technique.

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