Persistent Scatterer Density Improvement using Adaptive Deformation Models

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***Abstract*—Because the quality assessment of Persistent Scatterers (PS) is dependent on the deformation model chosen, PS may be falsely rejected due to model imperfections. To accept these PS, more advanced deformation models should be used. Two methods applying adaptive deformation models are proposed. The first is based on a sequential scheme of alternative hypothesis testing of extended deformation models within the integer least-squares framework. The second uses a iterative scheme of global deformation modeling based on previous PS results. Application of the techniques to a salt mining area in The Netherlands confirms the increase in the number of detected PS.**

I. Introduction

Persistent Scatterer Interferometry (PSI) techniques are powerful means to monitor deformation of the earth's surface. They rely on the identification of scatterers which remain coherent over the time interval under consideration. This identification is based on the assumption of ergodicity, either temporal or spatial. Assuming temporal ergodicity implies that the precision of a scatterer can be estimated by evaluating its behavior in time. This requires an a-priori deformation model. We refer to this model as the model under the null hypothesis, and it is used to unwrap the data in time and obtain a first estimation of the parameters of interest. Under the null hypothesis, we adopt a model for linear deformation, assuring maximum redundancy in the estimation problem.

Analysis of the residues (the deviations between the observations and the model) can identify both scattering noise (or *point noise*) as well as model imperfections, and is used to estimate the coherence of the scatterer. As a consequence, large residues due to model imperfections will result in type-I errors (false rejections) and coherent scatterers will be discarded for further analysis. In other words, Persistent Scatterers (PS) with more complex displacement histories will not be detected.

In this contribution two strategies are presented to increase the number of detected Persistent Scatterers using adaptive deformation models. The first strategy is based on alternative hypothesis testing during the PSI processing. A sequential scheme is used to apply and test extended deformation models per phase double difference, intending to find a model that sufficiently fits to the data to avoid type-I errors. The method is based on the integer least-squares technique, which allows the addition of extra deformation parameters without an increase of the computational burden. The adaption of the deformation model is based on hypothesis testing.

The second strategy is based on an iterative scheme. After finishing PSI processing under the null hypothesis, a global deformation model is estimated from the PS results. The model can be parametric (e.g., a subsidence bowl) or based on interpolation (e.g., Kriging). Further refinement of the deformation model used is obtained in an iterative scheme.

The strategies to increase the Persistent Scatterer density are applied to a salt mining area near Veendam in the Netherlands using ERS1/2 data. This area is mainly rural and therefore the number of objects that can potentially act as Persistent Scatterer is limited. The detection of as much Persistent Scatterers as possible is therefore crucial to obtain a detailed indication of the actual spatial deformation pattern.

II. Integer least-squares for PSI

The basic concept of the integer least-squares technique is to use the knowledge that some parameters, in this case the phase ambiguities, are integer valued. The problem can be formulated with the mathematical model

$$\begin{aligned} E\{y\} &= Aa + Bb, \quad y \in \mathbb{R}, a \in \mathbb{Z}, b \in \mathbb{R}; \\ D\{y\} &= Q_y, \end{aligned} \tag{1}$$

where $E\{.\}$ is the expectation operator, $D\{.\}$ the dispersion, y the vector of observations, A and B are design matrices for the integer and real valued parameter vectors a and b respectively, and Q_y is the covariance matrix. The system of equations (1) is solved in a three step procedure. First, the float solution is computed by neglecting the integer property of the ambiguities. Then, the ambiguities are resolved in a least-squares sense. To reduce the computation time, the ambiguities are decorrelated using the LAMBDA method (Least-squares AMBiguity Decorrelation Adjustment method) [1]. Finally, the float solution of the parameters of interest is updated using the fixed ambiguities.

The mathematical model (1) comprises of the functional and the stochastic model. The functional model describes the relation between the observations and the unknowns, whereas the stochastic model represents the statistical properties of the observations.

The functional model for a single master stack, where the master is indicated by a zero and N slave acquisitions are

available, has the form

$$E\{\begin{bmatrix} \psi^{01} \\ \vdots \\ \psi^{0N} \\ S^* \\ H^* \\ D_1^* \\ \vdots \\ D_P^* \end{bmatrix}\} = \begin{bmatrix} -2\pi & & \\ & \ddots & \\ & & -2\pi \\ & & \\ & & \\ & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} a^{01} \\ \vdots \\ a^{0N} \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{-4\pi}{\lambda} & \beta^{01} & \alpha_1(t^{01}) & \dots & \alpha_P(t^{01}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{-4\pi}{\lambda} & \beta^{0N} & \alpha_1(t^{0N}) & \dots & \alpha_P(t^{0N}) \\ 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix} \begin{bmatrix} S \\ H \\ D_1 \\ \vdots \\ D_P \end{bmatrix}, \quad (2)$$

where ψ are the phase observations, S is the atmospheric delay of the master acquisition, H the height, D_p are deformation parameters with $p = 1 \dots P$, λ is the radar wavelength, β is the height-to-phase conversion factor, α_p describes a deformation model as function of temporal baseline t and $(.)^*$ denotes a pseudo-observable needed to solve for the rank deficiency of the system [2]. The rank deficiency is caused by the fact that for each observed phase an ambiguity needs to be estimated, together with the parameters of interest. As a result, the number of unknowns exceeds the number of observations. With the introduction of pseudo-observables, the mathematical model is regularized.

The functional model (2) shows that in principle any number of deformation parameters can be used to estimate the deformation profile. However, with an increasing number of deformation parameters, the redundancy in the model decreases and thereby the stability of the estimation process. Therefore, the number of deformation parameters should be as low as possible, provided that the phase can still be unwrapped correctly. Examples of deformation models are a linear deformation rate, a higher order polynomial, a periodic signal or a breakpoint model [3]. A breakpoint model can be useful in case of a certain event during the analyzed time span, such as the start of oil or gas subtraction, or an earthquake. Obviously, different models can be combined or others can be designed based on a-priori knowledge about the deformation history in the area.

The second part of the mathematical model is the stochastic model, represented by the covariance matrix

$$D\{\begin{bmatrix} \psi \\ y^* \end{bmatrix}\} = \begin{bmatrix} Q_\psi & 0 \\ 0 & Q_{y^*} \end{bmatrix}, \quad (3)$$

where y^* represents the vector of pseudo-observations. The covariance matrix of the phase observations Q_ψ is obtained by variance component estimation (VCE) [4], [5]. The VCE technique estimates the covariance matrix directly from the data. Hence, the covariance matrix used in the estimation process is not dependent on a-priori assumptions on the quality of the data. After estimation and subtraction of certain signals, e.g. the atmospheric delay, the VCE algorithm is applied again to update the covariance matrix. The covariance matrix of the pseudo-observations Q_{y^*} contains variances which bound the solution space of the unknowns.

III. Adaptive deformation models method I: sequential hypothesis testing

The first method using adaptive deformation models is based on sequential hypothesis testing. The algorithm is initialized with the selection of a set of these models. Then, each phase double difference between two potential PS is unwrapped in time applying the sequential scheme of alternative hypothesis testing until a deformation model fits to the data well enough. A linear model is a good null hypothesis because of the maximum redundancy in the estimation process. The testing criterion for accepting a hypothesis is the a-posteriori variance factor

$$\hat{\sigma}^2 = \frac{e^T Q_\psi^{-1} e}{r}, \quad (4)$$

where e is the vector of residuals between the unwrapped phase and the deformation model and r is the redundancy in the functional model. A $\hat{\sigma}^2$ of 1.0 indicates that the covariance matrix used in the estimation process correctly describes the dispersion of the observations. Recall that this matrix was obtained by VCE. A value of 2.0 means that the stochastic model used is a factor two too optimistic (assuming that the functional model is correct). Hence, the a-posteriori variance factor scales the a-priori stochastic model for a specific double difference.

Applying the sequential scheme of hypothesis testing, a certain deformation model is accepted when $\hat{\sigma}^2$ is smaller than 1.0. Otherwise, the next model is tested until the complete set of models is evaluated. For computational efficiency, the sequential testing scheme is performed in batch, that is, a certain model is applied for a set of double differences, after which the next model is applied to a subset of these arcs which did not pass the test. For each arc the lowest $\hat{\sigma}^2$ and the corresponding model are stored. Then, even when the best model for an arc is not fitting well enough, the arc is accepted when the lowest $\hat{\sigma}^2$ is lower than a higher threshold, e.g. 3.0. A spatial procedure using a network of the arcs will finally test the correctness of the temporal unwrapping.

To reduce the computational burden, a PS distribution driven approach can be applied [3]. Here, the sequential scheme is directed by a prognosis of the PS density in a certain area based on amplitude dispersion.

IV. Results sequential hypothesis testing

The sequential hypothesis testing procedure is applied to a single master stack of ERS1/2 images covering an area near Veendam in the north part of The Netherlands. The area, which is mainly rural, experiences subsidence due to salt extraction (since 1995). The salt extraction results in a bowl shaped

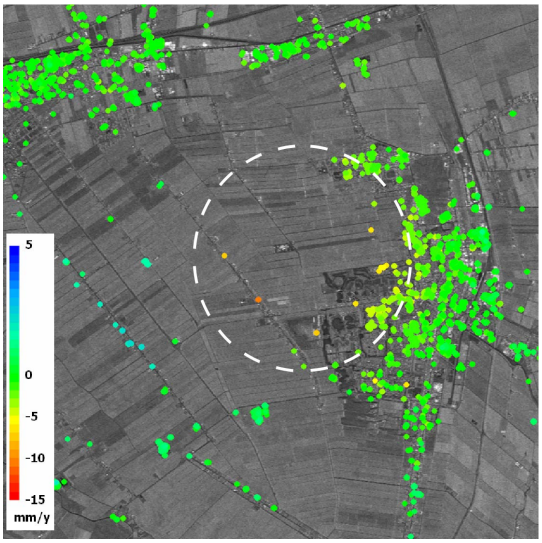

Fig. 1. PSI result using a linear deformation model for a salt mining area near Veendam, The Netherlands (white circle). The analysis is based on 63 ERS1/2 images acquired between May 1992 and January 2005.

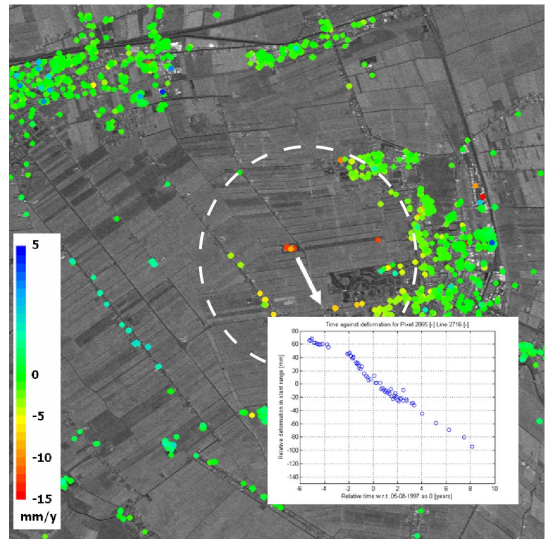

Fig. 2. PSI result obtained by applying the sequential hypothesis testing scheme using a linear and a breakpoint model. Linear deformation rates estimated from the unwrapped time series are shown. The breakpoint was set a-priori at 22 May 1995 based on the start of the salt extraction. The inset shows the displacement profile in the center of the subsidence area.

deformation signal. A total of 63 images covering the period from May 1992 to January 2005 are used. The results of standard processing using a linear deformation model is shown in Fig. 1. The contours of the subsidence bowl are visible in the village. However, no PS are detected in the center of the subsidence bowl. Yet, the occurrence of buildings and other man-made features in the area suggest a potential for PS.

Fig. 2 shows the result after applying the sequential testing scheme using a linear and a breakpoint model. The breakpoint is defined a-priori at 22 May 1995, based on the start of the salt extraction. With this model PS are detected in the center of the subsidence region. To enable visualization, the figure shows linear deformation rates estimated through the unwrapped time series, even when the breakpoint model was used for the unwrapping. The inset shows the actual displacement profile of these PS. The additionally detected PS are of paramount importance for the reliable estimation of the spatial deformation pattern due to salt extraction in Veendam. They make the difference between applicability and non-applicability of PSI in this case. Note however that the result using adaptive deformation models contains more outliers, which could be autonomous moving PS or falsely accepted PS (type-II errors).

V. Adaptive deformation models method II: iterative deformation modeling

The second method is based on an iterative scheme of deformation modeling. After a standard PSI processing under the null hypothesis, that is, applying a linear model, a deformation model is estimated from the PS results. The modeled deformation is then subtracted from the original interferometric phase and the PSI processing is repeated (again using a linear model). Because the deformation models are estimated per epoch using the displacement time series, possible non-linear deformation is modeled as well. As a result, points which were previously rejected as a PS due to too large deviations from the model may now be accepted. Hereby the density of PS improves. Obviously, this procedure can be repeated iteratively. Note that a similar procedure is often followed in standard PSI processing to remove atmospheric delays and, based on auxiliary data, heights.

The deformation modeling can either be parametric or based on interpolation (e.g. Kriging). An example of a parametric representation is an ellipsoidal subsidence bowl

$$D_{\text{mod}} = d \exp(-\frac{1}{2}(u^2 + v^2)), \tag{5}$$

where

$$\begin{aligned} u &= \frac{((x - x_c)\sin(\theta) + (y - y_c)\cos(\theta))^2}{r_1^2} \\ v &= \frac{((x - x_c)\cos(\theta) - (y - y_c)\sin(\theta))^2}{r_2^2}. \end{aligned}$$

Here D_{mod} is the modeled deformation, d is a scaling factor, x and y are the azimuth and range coordinates of the PS, x_c and y_c are the center coordinates of the subsidence bowl, r_1 and r_2 are the long and short axis and θ is the orientation of the ellipsoid. To estimate the parameters of the non-linear function in a least-squares sense, approximate values of the parameters are required. In practice these approximate values need to be known accurately to ensure convergence to a solution. In case the subsidence phenomenon can be modeled by a circular bowl, the parameterization (5) simplifies significantly.

An alternative for parametric modeling is an interpolated deformation field, e.g. obtained by Kriging. Kriging uses a spatial covariance function to predict a signal at locations where the signal is unknown. The covariance function is

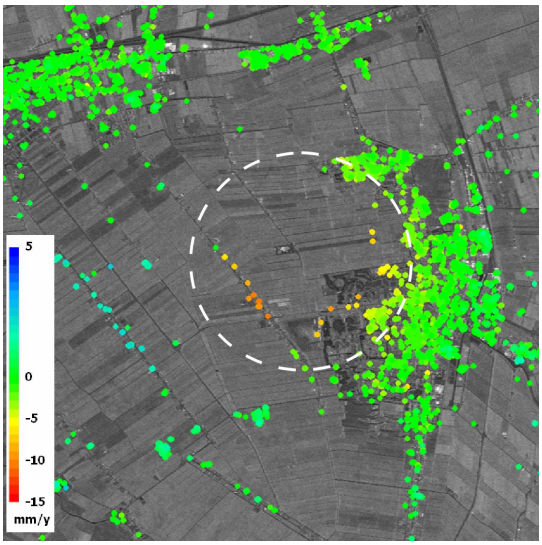

Fig. 3. PSI result using a linear deformation model for a salt mining area near Veendam, The Netherlands (white circle). The analysis is based on 63 ERS1/2 images acquired between May 1992 and January 2005.

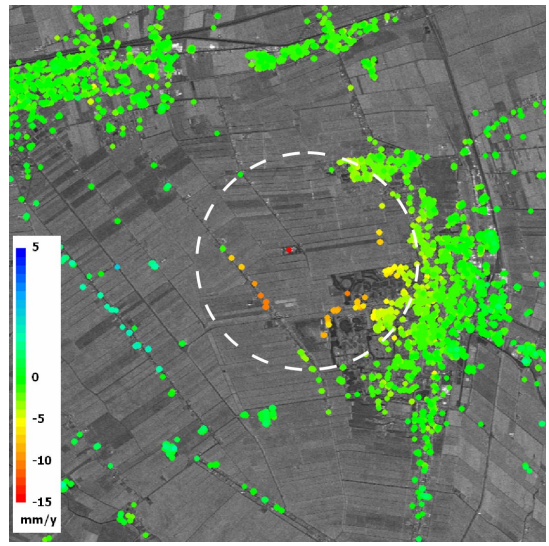

Fig. 4. PSI result obtained the iterative deformation modeling method using Kriging. Linear deformation rates estimated from the unwrapped time series are shown.

estimated per epoch from the data and does not require a-priori information about the deformation field. This method is therefore more flexible than the parametric representation and widely applicable.

VI. Results iterative deformation modeling

The iterative deformation modeling method is applied to the Veendam area using Kriging to model the deformation field. The result of standard processing is shown in Fig. 3. These results are used for the deformation modeling by Kriging. Fig. 5 shows an example of the predicted deformation field for a single epoch at the end of the time series (maximum deformation). The deformations are predicted on locations with relatively low amplitude dispersion. Even though no PS were detected in the center of the subsidence region, Fig 5 shows that the subsidence bowl could still be predicted. The detected PS after one iteration of deformation modeling is shown in Fig. 4. The number of detected PS increased from 13074 to 14963, confirming the expected increase in PS density. Moreover, the modeling enabled the detection of a PS in the center of the subsidence region.

VII. Conclusions

Because the quality assessment of PS is dependent on the deformation model chosen, PS are falsely rejected due to model imperfections (type-I errors). To detect these PS, adaptive deformation models should be used. Two methods are proposed. Both the sequential hypothesis testing scheme and the iterative deformation modeling approach show an increased density of PS. Application of the iterative modeling approach to a salt mining area in The Netherlands shows an increase in the number of detected PS from 13074 to 14963. Moreover, both methods enabled the detection of PS in the center of the subsidence area, which were not detected using standard processing.

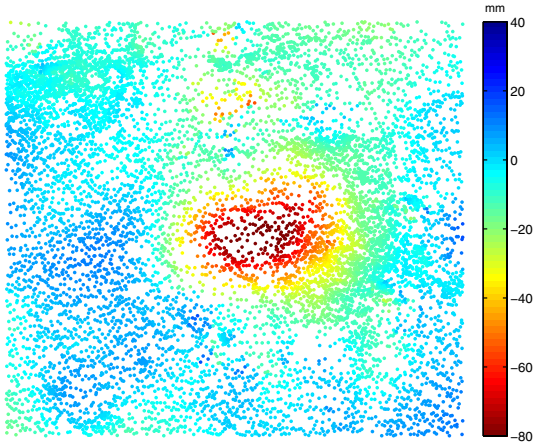

Fig. 5. Example of the predicted deformation field [mm] for a single epoch at the end of the time series (maximum deformation). The deformations are predicted on locations with relatively low amplitude dispersion.

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