Recursive data processing and data volume minimization for PS-InSAR

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***Abstract*— PS-InSAR has proven to be an accurate and efficient technique for the joint estimation of topographic and displacement signal from stacked interferometric combinations. In this contribution a new method for PS-Insert processing is introduced, which enables the recursive estimation of parameters of interest. The method is based on the ILSQ PS-InSAR concept and makes use of the estimation vector and corresponding variance-covariance matrix of the initial estimation epoch. The presented methodology systematically adds a new acquisition (or set of acquisitions) to the existing stack, updates the solution of the previous run, and analyzes whether the behaviour of the (pre-) selected points fits the expected one. This contribution focuses on a mathematical framework, rather then on specific applicational problems. Nevertheless, the performed numerical analysis on simulated data sets is analyzed and discussed, which shows that the preset aims of the recursive PS-InSAR estimation technique is achieved.**

I. Introduction

Time series InSAR analysis using persistent scatterer (PS) techniques aims at the joint estimation of topographic and displacement signal from a number of interferometric combinations, [1], [2]. Since the estimates of both parameters are correlated and error signal due to, e.g., atmospheric signal can significantly affect the adjustment, an accurate estimation depends on the availability of a large data stack, i.e., more than 20-30 images. A smaller number of images usually results in problems like detecting the potential PS, reducing the atmospheric signal, separating topography and displacement, and phase ambiguity estimation.

An additional problem for all current multi-image processing concepts is that the parameter estimation is usually performed in batches, i.e., by using all available acquisitions at once. Hence, in order to incorporate a newly available acquisition into the processing chain, and consequently update the estimates, the whole processing (at least the PS part) has to be performed again. Such an approach consequently leads to an increase of processing time, limits the application to the areas where only a sufficient number of images is available, and reduces the potential application of the method to a semi-real-time deformation monitoring.

The two main processing concepts of PS-InSAR are the concept of the ambiguity function, [1], and Integer Least Squares (ILSQ) method, [2]. The main drawback of the first one is that the propagation concept of observations to the unknown parameters is suboptimal. Moreover, the method strongly depends on the discretization of the solution space and it treats unknown ambiguities as deterministic parameters instead of stochastic ones. The ILSQ approach is based on the principles of Best Linear Unbiased Estimation (BLUE) – it is based on the minimization of the mean squared error and it is formulated as a constrained minimization problem on the integer nature of the unknowns, [6]. By means of the ILSQ method, the quality description of estimated parameters is the one of the end products of the analysis, which can consequently be used to determine the significance and reliability of the estimated parameters.

The ILSQ PS-InSAR processing framework sets the basis for a recursive data processing strategy, where new acquisitions can be easily added to an existing data stack, significantly reducing the computational requirements. This implies that the presented methodology systematically adds a new acquisition to the existing stack, updates the solution of the previous run, and analyzes whether the behaviour of the (pre-)selected points fits to the expected behaviour of parameters of interest. If not, an alternative hypothesis is tested against the prior solution, leading to the rejection of the point, adaptation of the model, or manual intervention.

For the conditions on the practical application of recursive PS-InSAR processing, it can be referred to the block–diagonal structure of the variance-covariance matrix of the introduced recursive model (the estimates from the initialization run and phase observations of the additional acquisition are assumed to be uncorrelated). Secondly, the atmospheric and non–modelled displacement contributions to the interferometric phase have to be modelled and incorporated into the variance matrix by means of covariance functions, [4], [5] – in the presented study the covariance functions are not further elaborated on. Moreover, in numerical experiments, phase contributions are isolated by low–pass filtering in the spatial domain and high–pass filtering in the temporal domain. Furtheron, in order to correctly perform the initialization run (candidate selection and unwrapping), a sufficient number of images (15–20) is needed.

In the following sections the concept of the recursive PS-InSAR is presented. Examples on simulated data are used

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to illustrate intermediate and final products and validate the concept.

II. Recursive ILSQ PS-InSAR: Analysis

A. ILSQ for PS-InSAR

The general mathematical model for PS-InSAR in Gauss-Markov formulation has the following form:

$$E\{\begin{bmatrix}\underline{y}_1\\ \underline{y}_2\end{bmatrix}\} = \begin{bmatrix}A_1 & B_1\\ A_2 & B_2\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}; \quad D\{\begin{bmatrix}\underline{y}_1\\ \underline{y}_2\end{bmatrix}\} = \begin{bmatrix}Q_{y_1} & 0\\ 0 & Q_{y_2}\end{bmatrix}, \tag{1}$$

where: $\underline{y}_1$ stands for double-difference phase observations, $\underline{y}_2$ for pseudo observations, $A_{1,2}$ and $B_{1,2}$ are design matrices, a ambiguities ($a \in \mathbb{Z}$), and b unknown parameters of interest ($b \in \mathbb{R}$). In order to simplify notation $A_{1,2}$ are denoted as A and $B_{1,2}$ as B. Note that in the formulated model, the pseudo-observables are required to solve the model rank-deficiency.

The BLUE of x of a constrained linear model, Eq. (1), is obtained in two steps. First, the BLUE estimate of x is obtained from the unconstrained linear model $E\{\underline{y}\} = [A, B]x$, resulting with a float solution $\underline{\hat{x}} = [\hat{a}, \hat{b}]^T$ and corresponding variance matrix $Q_{\hat{x}}$. This result is then input for the second step, where the BLUE of x for constrained model is obtained as the BLUE of $E\{\underline{\hat{x}}\}$ giving a fixed solution of the unknowns. Hence, the unknowns of the constrained model are determined by, [3]:

$$\underline{\check{b}} = \underline{\hat{b}} - Q_{\hat{b}\hat{a}}Q_{\hat{a}}^{-1}(\hat{a} - \check{a}) \tag{2}$$

with corresponding variance matrix:

$$Q_{\check{b}} = Q_{\hat{b}} - Q_{\hat{b}\hat{a}}Q_{\hat{a}}^{-1}Q_{\hat{a}\hat{b}} \ . \tag{3}$$

In a practical application of ILSQ to PS-InSAR the upper described procedure is solved by means of a three-step procedure, where an extra step is introduced to resolve the ambiguities. This addition is related to an optimization of the ambiguity search spaces and it is realized through the *Least-squares AMBiguity Decorrelation Adjustment method* (LAMBDA) algorithm, [2], [3]. This optimization results in a fixed solution for the ambiguities $\check{a}$. The fixed ambiguities are consequently used to obtain the solution of the vector of unknown parameters $\check{b}$, by means of Eqs. (2) and (3)

For more details on the stochastic part of the observation model, Eq. (1), reader is referred to the concepts presented in [5] and [4].

B. Recursive ILSQ PS-InSAR

The presented mathematical ILSQ for PS-InSAR framework serves as a basis for recursive PS-InSAR processing. The goal of recursive PS-InSAR is to estimate the parameters of interest recursively from the observed data – double difference interferometric phase observations. The starting point in the analysis is a partitioned model, formed of the solution of the estimates from previous epochs and observation equations of new observations:

$$\begin{aligned} E\{\begin{bmatrix}\underline{\check{x}}_{i|k}\\ \underline{y}_k\end{bmatrix}\} &= \begin{bmatrix}I & 0\\ A_k & -2\pi\end{bmatrix} x_{k|k} \overset{k=1}{=} \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ f_t & f_{B_{perp}} & -2\pi\end{bmatrix}\begin{bmatrix}v_{k|k}\\ h_{k|k}\\ a_{k|k}\end{bmatrix}\\ D\{\begin{bmatrix}\underline{\check{x}}_{i|k}\\ \underline{y}_k\end{bmatrix}\} &= \begin{bmatrix}Q_{\check{x}_{i|k}} & 0\\ 0 & Q_k\end{bmatrix} \overset{k=1}{=} \begin{bmatrix}Q_{\check{x}_{i|k}} & 0\\ 0 & \sigma_{y_k}^2\end{bmatrix}, \end{aligned} \tag{4}$$

where parameters related to epoch i prior to new epoch k are $\underline{\check{x}}_{i|k} = [\check{v}_{i|k}, \check{h}_{i|k}]^T$ representing estimates of displacement parameters and residual height w.r.t. the reference surface (ellipsoid or a-priori DEM), with corresponding variance-covariance matrix $Q_{\check{x}_{i|k}}$. Parameters related to the new measurement are the ambiguity of the phase observation, a_k, which is again resolved by means LAMBDA method. f_t and $f_{B_{perp}}$ stand for temporal baseline and height-to-phase conversion factor, [4]; while σ_{y_k} stands for standard deviation of the new phase measurement, [5].

The mathematical proof of the equality of batch and recursive estimation is given in the Appendix. The appendix equation set shows that there is no need to store the previous observables $\underline{y}_i$, $i = 1, \ldots, k-1$ (e.g. $k-1$ interferograms and PS phase history), for the purpose of computing the present least-squares estimator $\underline{\check{x}}_{k|k}$. That is, the estimator $\underline{\check{x}}_{k|k}$ can be computed directly from the previous estimator $\underline{\check{x}}_{i|k}$, its corresponding variance matrix $Q_{\check{x}_{i|k}}$, and the present observable y_k. This is the essence of recursive estimation. Therefore it is obvious that the recursive estimation procedure has to be initialized with the computation of the initial least-square estimator. In this case a solution is denoted as $\underline{\check{x}}_{i|k}$ and is computed with Eqs. (2) and (3).

Once $\underline{\check{x}}_{i|k}$ is known, $\underline{\check{x}}_{k|k}$ can be computed from $\underline{\check{x}}_{i|k}$ and $\underline{y}_k$ using the following equation set (for derivation see Appendix):

$$\begin{cases}\underline{\check{x}}_{k|k} = \underline{\check{x}}_{i|k} + Q_{\check{x}_{k|k}}A_k^TQ_k^{-1}(\underline{y}_k - A_k\underline{\check{x}}_{i|k})\\ Q_{\check{x}_{k|k}} = (Q_{\check{x}_{i|k}}^{-1} + A_k^TQ_k^{-1}A_k)^{-1} \ .\end{cases} \tag{5}$$

The first equation is referred to as the *measurement update equation* (MU). It clearly shows how to update the previous estimator $\underline{\check{x}}_{i|k}$ in order to take care of the new observable $\underline{y}_k$. The second equation is called the *variance update equation* (VU).

C. Recursive processing update equations: discussion

In the MU equation, the vector $A_k\underline{\check{x}}_{i|k}$ is referred to as a correction term. This vector depends on all previous observables $\underline{y}_i$, $i = 1, \ldots, (k-1)$, but it is independent of the current one $\underline{y}_k$. Furtheron, since $\underline{\check{x}}_{i|k}$ is unbiased, $E\{\underline{\check{x}}_{i|k}\} = x$, $A_k\underline{\check{x}}_{i|k}$ is an unbiased estimator of $E\{\underline{y}_k\} = A_kx$. In fact, $A_kE\{\underline{\check{x}}_{i|k}\}$ is a BLUE of $\underline{y}_k$ when the estimation is based on the partitioned model in Eq. (9). Hence, $A_k\underline{\check{x}}_{i|k}$ can be interpreted as the prediction of the present observable $\underline{y}_k$. The difference $\underline{y}_k - A_k\underline{\check{x}}_{i|k}$ in Eq. (5) is therefore the residual value between the observation and its prediction. This difference is called the *prediction residual* and denoted as:

$$\underline{\nu}_k = y_k - A_k\underline{\check{x}}_{i|k} \ . \tag{6}$$

In Eq. 5, the predicted residual $\underline{\nu}_k$ is premultiplied by the *gain matrix*:

$$K_k = (Q^{-1}_{\check{x}_{i|k}} + A_k^T Q_k^{-1} A_k)^{-1} A_k^T Q_k^{-1} \tag{7}$$

and the product is then added to the previous estimator $\underline{\check{x}}_{i|k}$ to obtain the current estimation $\underline{\check{x}}_{k|k}$. Hence, the gain by including the observable y_k is determined by product $K_k \underline{\nu}_k$.

Equation (5) also shows that the correction to the estimate $\underline{\check{x}}_{i|k}$ is small if the predicted residual $\underline{\nu}_k$ is small. If the predicted observation $A_k \underline{\check{x}}_{i|k}$ is close to the actual observation y_k there is no need to change the estimate $\underline{\check{x}}_{i|k}$ by a large amount. Furthermore, Eq. (5) also shows that the correction to the estimate $\underline{\check{x}}_{i|k}$ is small if the elements in the gain matrix K_k are small. From Eq. (7) it is clear that the gain matrix K_k depends on the precision of the previous estimator $Q_{\check{x}_{i|k}}$, and the precision of the added phase measurement Q_k.

D. Hypothesis testing in the case of recursion

The standard feature of the ILSQ PS-InSAR estimation algorithm is that once estimates of unknown parameters and corresponding variance matrix are obtained, the validity of the mathematical model is tested on errors in observations, design matrix, and variance matrix, [2]. It is possible to derive a recursive procedures for hypothesis testing, namely the Overall Model Test (OMT), [7].

In a similar manner, like in the case of the recursive estimator, a scheme for a recursive update of the $\underline{\hat{e}}^T Q_y^{-1} \underline{\hat{e}}$ teststatistic can be derived. For more details on the derivation, see [7]. Adopting the notation of introduced partitioned model, Eq. (4), the recursive update of OMT teststatistic is realized by means of:

$$\begin{cases} (\underline{\hat{e}} Q_y^{-1} \underline{\hat{e}})_{k|k} = (\underline{\hat{e}} Q_y^{-1} \underline{\hat{e}})_{i|k} + \underline{\hat{\nu}}^T_{k|k} Q_{\nu_{k|k}} \underline{\hat{\nu}}_{k|k} \\ Q_{\nu_{k|k}} = (Q_{k|k} + A_k Q_{\underline{\check{x}}_{i|k}} A_k^T) \ . \end{cases} \tag{8}$$

The recursive OMT teststatistic update shows how the norm of the least-squares residual vector should be updated when new observables become available. This approach enables us to test whether a point which was assumed to be stable in the past, remains stable when a new set of acquisitions becomes available. However, due to the wrapped nature of the observables, the standard hypothesis testing procedures may accept the null hypothesis, even if the new point is not stable. Hence, the testing algorithms for recursive PS-InSAR need to further optimised and developed. This will be a study of the future work.

E. Argumentation for recursive data processing

Both processing strategies are capable of incorporating observations stemming from phase measurements as well as those from the system knowledge, i.e., stochastic nature of observables. Although the processing strategy differs, the estimation principle does not. Therefore, an identical estimator $\underline{\check{x}}_{k|k}$ for the parameter estimation at epoch k will be obtained by using both concepts, see the Appendix.

In a batch estimation, the complete vector of observations is processed at once and all unknowns are estimated together. In this case, the realization of the data processing can be hindered by practical limitations (computer memory restrictions and/or data availability). In a recursion the PS-InSAR estimation per epoch will have a relatively small size. Original images and pre-processed data can be removed from the processing system once they are processed and do not need to be stored. However, in order to start with the PS-InSAR recursion, the initial estimate $\underline{\check{x}}_{i|k}$ of parameters of interest, $i \geq 15$ acquisitions, has to be available, e.g., a batch solution for the first epoch (initialization) has to be used.

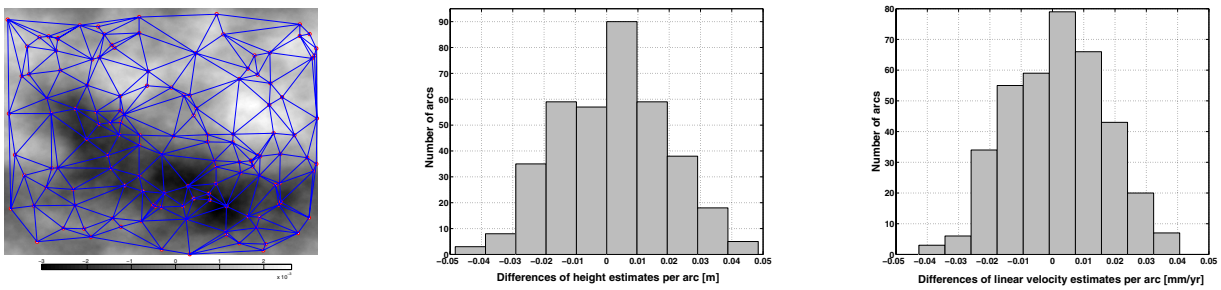
Moreover, estimations in batch can provide results with a delay of a couple of years, e.g., after enough images are acquired, delivered, pre-processed, etc. However, the recursion can be run parallel to data gathering. Furtheron, with appropriate covariance models, for atmosphere and non-linear deformation, estimates of the parameters of interest can be available in near real-time.

The main disadvantage of the recursive PS-InSAR is related to the hypothesis testing procedures. This is because, in the recursion, previous observables are removed and only the final solution is stored and updated. Nevertheless, with a further algorithmical and applicational optimisation of the recursive PS-InSAR approach, i.e., by developing dedicated testing procedures, this drawback can be circumvented. Another limitation is related to the unavailable time series of phases of the PS, since the non–model displacement and atmospheric phase screen are removed from an observed phase by means of a low–/ high–pass filtering approach by utilizing the full phase history of PS. This limitation can also be eliminated by developing a class of covariance functions to model these error sources, and including them into the stochastic part of the model. Both of these drawbacks will be the topics of further research.

III. Simulational and implementational analysis

In order to get a further insight in the performance and complexity of the recursive PS-InSAR algorithm, an analysis on simulated data is performed. A number of test runs under different conditions (e.g., different error sources, geometries, etc.) are performed and analyzed. In general, the simulated interferometric phase follows the model of [4], with the following parameters: the ERS type sensor; phase accuracy $\sigma_\varphi = 0.06[cyc]$; orbital tube of $B_{perp} = 500[m]$ with sensor localization precision of $10[cm]$; APS is simulated as a superposition of two fractal regimes $5/3$ and $8/3$, [4]; the topography is simulated with $\sigma_{topo} = 10m$; the velocity field with $\sigma_{d1} = 0.001[m]$ as a measure of the expected linear deformation and $\sigma_{d2} = 0.02[m]$ for the expected periodic deformation. Furthermore, for a non-linear deformation filtering a (temporal) block filter of one-year length is applied.

Here, a test run on a data stack consisting of 30 (simulated) interferograms is discussed. The test strategy is that first arc-estimates from batch processing are obtained (using all 30 interferograms at once), see Figure 1(a). Then, the second estimation is performed in the recursive manner, first, a batch 24 interferograms solution is obtained, and subsequently updated



(a) PS network (Delauney triangulation of PSs) overlaid on top of the simulated deformation field.

(b) Histogram of (per arc) differences of estimated heights from batch and recursive ILSQ PS-InSAR strategies. The differences are normally distributed with zero mean and standard deviation of $\sigma_{\Delta H} = 0.02\ m$.

(c) Histogram of (per arc) differences of estimated velocities from batch and recursive ILSQ PS-InSAR strategies. The differences are normally distributed with zero mean and standard deviation of $\sigma_{\Delta H} = 0.015\ mm/yr$.

Fig. 1. Results of numerical analysis. In plots (a) and (b) vertical axis stands for a number of arcs $[0-90]$, while a horizontal one for differences in estimates per arc $[-0.05-0.05]$ in $[m]$ and $[mm/yr]$ respectively.

with the remaining 6 interferograms. The updating strategy was that in the new epochs the number of interferograms was enlarged by one, e.g., $1+2+3$.

The results obtained from two different estimation procedures show a significant degree of match. The empirical validation, see Figures 1(b) and 1(c), shows that the difference between the two processing strategies is insignificant with respect to the input simulation parameters. Moreover, both differences are normally distributed with zero mean and standard deviations of $\sigma_{\Delta H} = 0.02[m]$ and $\sigma_{\Delta Vel} = 0.015[mm/yr]$ respectively.

IV. Concluding Remarks

A mathematical framework for the recursive PS-InSAR processing has been developed and tested on simulated data. The essence of the recursive PS-InSAR approach is that there is no need to store past measurements (i.e., interferograms) for the purpose of computing the updated least-squares estimates. Furthermore, recursion in PS-InSAR framework gives a significant improvement in processing time by means of an efficient computation of the corresponding best estimates.

Appendix

Consider a *partitioned model*:

$$E\{\begin{bmatrix}\underline{y}_i\\ \underline{y}_k\end{bmatrix}\} = \begin{bmatrix}A_i\\ A_k\end{bmatrix}x; \quad D\{\begin{bmatrix}\underline{y}_i\\ \underline{y}_k\end{bmatrix}\} = \begin{bmatrix}Q_i & 0\\ 0 & Q_k\end{bmatrix}. \tag{9}$$

Note that it is assumed that the measurement epochs $\underline{y}_i$ and $\underline{y}_k$ are mutually uncorrelated. The least-squares solution of Eq. (9) will be denoted as $\underline{\check{x}}_{k|k}$. It reads:

$$\begin{cases}\underline{\check{x}}_{k|k} = (A_i^T Q_i^{-1} A_i + A_k^T Q_k^{-1} A_k)^{-1} \cdot \\ \qquad\qquad (A_i^T Q_i^{-1}\underline{y}_i + A_k^T Q_k^{-1}\underline{y}_k) \\ Q_{\check{x}_{k|k}} = (A_i^T Q_i^{-1} A_i + A_k^T Q_k^{-1} A_k)^{-1} \quad .\end{cases} \tag{10}$$

Let us now consider the *partial model*:

$$E\{\underline{y}_i\} = A_i x; \quad D\{\underline{y}_i\} = Q_i \quad . \tag{11}$$

Its solution will be denoted as $\underline{\check{x}}_{i|k}$. It reads:

$$\begin{cases}\underline{\check{x}}_{i|k} = (A_i^T Q_i^{-1} A_i)^{-1}(A_i^T Q_i^{-1}\underline{y}_i) \\ Q_{\check{x}_{i|k}} = (A_i^T Q_i^{-1} A_i)^{-1} \quad .\end{cases} \tag{12}$$

From this it follows that:

$$(A_i^T Q_i^{-1} A_i) = Q_{\check{x}_{i|k}}^{-1} \quad \text{and} \quad A_i^T Q_i^{-1}\underline{y}_i = Q_{\check{x}_{i|k}}^{-1}\underline{\check{x}}_{i|k} \ . \tag{13}$$

Substitution of this result into the solution of the partitioned model, Eq. (10), shows that:

$$\begin{cases}\underline{\check{x}}_{k|k} = (Q_{\check{x}_{i|k}}^{-1} + A_k^T Q_k^{-1} A_k)^{-1}(Q_{\check{x}_{i|k}}^{-1}\underline{\check{x}}_{i|k} + A_k^T Q_k^{-1}\underline{y}_k) \\ Q_{\check{x}_{k|k}} = (Q_{\check{x}_{i|k}}^{-1} + A_k^T Q_k^{-1} A_k)^{-1} \quad .\end{cases} \tag{14}$$

This is identical to the solution of the model:

$$E\{\begin{bmatrix}\underline{\check{x}}_{i|k}\\ \underline{y}_k\end{bmatrix}\} = \begin{bmatrix}I\\ A_k\end{bmatrix}x; \quad D\{\begin{bmatrix}\underline{\check{x}}_{i|k}\\ \underline{y}_k\end{bmatrix}\} = \begin{bmatrix}Q_{\check{x}_{i|k}} & 0\\ 0 & Q_k\end{bmatrix}. \tag{15}$$

Hence, it is proven that the solution of the partitioned model Eq. (9) can be found in two steps. In order to obtain the estimates for $\underline{\check{x}}_{i|k}$ and $Q_{\check{x}_{i|k}}$, first the partial model Eq. (11) is solved. Then in a second step $\underline{\check{x}}_{i|k}$ and $Q_{\check{x}_{i|k}}$ together with $\underline{y}_k$ and Q_k are used to find the final solution, via model Eq. (15). This result shows that there is no need to store past observations y_i for the purpose of computing the updated estimator $\check{x}_{k|k}$ and $Q_{\check{x}_{k|k}}$. Note that rearrangement of the right-hand side of the first equation of Eq. (14) leads to the form of Eq. (5). The presented derivation can be easily generalized to two, or more then two, recursion steps by defining i and k as running indices in a certain range with $i < k$ condition.

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