RECURSIVE PERSISTENT SCATTERER INTERFEROMETRY

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ABSTRACT

This paper presents an initial sketch for a recursive Persistent Scatterers Interferometry (PSI) processing, which enables the sequential estimation of parameters. The method is based on the Integer Least Squares (ILSQ) PSI concept and makes use of the estimation vector and corresponding variance-covariance matrix of the initial estimation epoch. In addition, the concept of multi–modal adaptive estimation and testing is applied. The presented methodology systematically adds a new acquisition or set of acquisitions to the existing stack, updates the solution of the previous run, and analyzes whether the behavior of the (pre-) selected points fits the expected one. Here we focus on the mathematical framework, rather then specific application problems. Nevertheless, the performed numerical analysis on simulated data sets is elaborated and discussed, which shows that the preset aims of the recursive PSI estimation technique, i.e., the sequential update of the paramaters without storing all the data and the recursive unwrapping, are achieved.

1 INTRODUCTION

Persistent Scatterer Interferometry (PSI) technique aims at the joint estimation of topographic and displacement parameters from a number of interferometric combinations, [1, 2]. Since the estimates of these parameters are correlated and error signal due to, e.g., atmospheric signal can significantly affect the adjustment, an accurate estimation usually relies on the availability of a large data stack [1]. A too limited number of images usually results in problems related to phase ambiguity estimation, detecting the potential PS, reducing the atmospheric signal, topography and displacement separation.

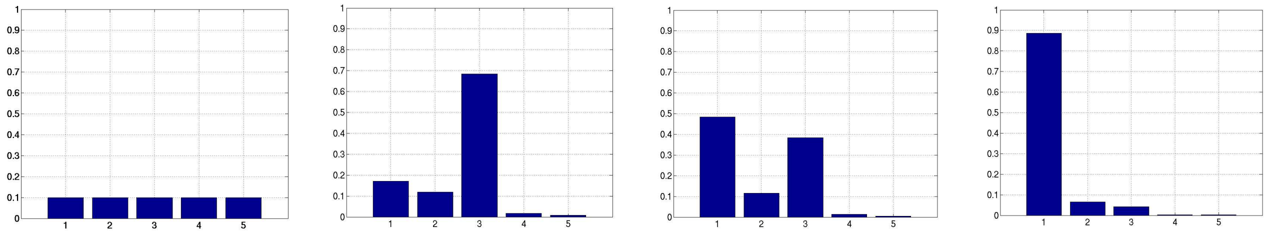
An additional problem for all current multi-image processing algorithms is that the parameter estimation is usually performed by using all available acquisitions at once, i.e., in batch. Therefore, in order to incorporate a newly available acquisition into the estimation process, and consequently update the estimates, the whole PSI processing has to be performed again. Such an approach consequently leads to a cumulative increase in processing time, it limits the application to those areas where only a sufficient number of images is available, and it reduces the potential application of the method to a semi-real-time deformation monitoring. Finally, when a new sensor is launched, it may take several years before an analysis can be started.

The primary goal of this study is the development and validation of a new methodology for recursive PSI. The secondary goal is the development of a recursive PSI phase unwrapping system. The new methodology must: (i) accurately and quickly perform the ambiguity resolution with limited data availability, and (ii) provide estimates of the parameters of interest including their quality description (usually deformation and residual height) at all measurement epochs of the interferometric stack sequence.

1.1 Problem Definition

The problem of PS ambiguity resolution generally consists of two main functions, identifying possible ambiguity sets and determining which one of them is correct. Numerous efficient methods have been developed to tackle the first part of the ambiguity resolution problem, and it is not the primary focus of this research.

From PSI perspective, most current methods, e.g., [1, 2], simply "wait" until enough information has been obtained, and then derive a solution by systematically sampling the solution space, until the correct ambiguity set becomes evident. The disadvantage of this "off-line" approach is, that it ignores the possibility of attaining a solution including quality description before the ambiguities are declared "fixed". Some new efforts have explored the possibility of simply using the current "best" ambiguity vector at each time epoch of interferometric stack sequence, even if it has not yet been validated as the correct set, [3]. Although this method may gave satisfactory results, e.g., for the case when the initialization is performed with 10–15 images and under the assumptions of linearity for the deformation phenomena, the problems regarding phase unwrapping emerged. The main disadvantage of this approach is that it relies only on the floating filter and ignores information available from other ambiguity set solutions, as well as the time history of the candidate ambiguity sets, see Fig.1.



(a) IFGS: +1 (b) IFGS: +5 (c) IFGS: +10 (d) IFGS: +15

Figure 1: Histogram plots of multi-modal probability density functions of different ambiguity resolution vectors as a function of a number of stacked interferograms. Note that the correct solution is the vector one and that if only one interferogram is available, all solutions have equal probability of being the correct one, case (a). If a small number of images is available or the observed signal significantly deviates from linear behavior, the incorrect solution will be estimated, case (b). As a consequence of stacking more information probability density function of the correct solution is indicated in time, cases (c) and (d).

The work presented here is an extension of [3], using recursive estimation techniques and Multi-Modal Adaptive Estimation (MMAE), [4]. Within this "on-line" concept, the measurement sequence is analyzed in near real time to data gathering, where probability of each of the ambiguity candidates is computed and evaluated through belief networks, see [5].

The basic idea of the algorithm is a dynamic processing and construction of the belief network, based on the previous estimates. This idea is realized through a set a finite number of estimators, each of which hypothesizes a different ambiguity set. The residual vector and its covariance, computed as the difference between the measurement and the filter's prediction of the measurements before they arrive, are then evaluated by hypotheses testing. The output of testing gives a relative indication of how close is each of the ambiguity sets (that are used in different filters) is to the "true" ambiguity model. This processing concept enables the recursive computation of belief networks - where the computation of belief of every ambiguity node (its conditional probability) given the statistical evidence that has been observed so far, is computed recursively.

Section 2 presents the theoretical framework, recursive estimation, multiple filtering, and PSI ambiguity resolution theory used to develop the mathematical models for this research. Section 3 gives details of the flow of the new recursive algorithm. Concepts necessary for understanding the output of the overall algorithm are also discussed. Section 4 presents a summary and conclusions and recommendations for future work.

2 THEORY DEVELOPMENT

2.1 Integer Least Squares (ILSQ) for PSI

The general mathematical model for PSI in the Gauss-Markov formulation has the following form:

$$E\{\begin{bmatrix}\underline{y}_1\\ \underline{y}_2\end{bmatrix}\} = \begin{bmatrix}A_1 & B_1\\ A_2 & B_2\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}; \quad D\{\begin{bmatrix}\underline{y}_1\\ \underline{y}_2\end{bmatrix}\} = \begin{bmatrix}Q_{y_1} & 0\\ 0 & Q_{y_2}\end{bmatrix}, \tag{1}$$

where: $\underline{y}_1$ stands for double-difference phase observations, $\underline{y}_2$ for pseudo observations, $A_{1,2}$ and $B_{1,2}$ are design matrices, a ambiguities ($a \in \mathbb{Z}$), and b unknown parameters of interest ($b \in \mathbb{R}$). In order to simplify notation $A_{1,2}$ are denoted as A and $B_{1,2}$ as B. Note that in the formulated model, the pseudo-observables are required to accommodate for the model rank-deficiency.

The BLUE of x of a constrained linear model, Eq. (1), is obtained in two steps. First, the BLUE estimate of x is obtained from the unconstrained linear model $E\{\underline{y}\} = [A, B]x$, resulting in a float solution $\underline{\hat{x}} = [\hat{a}, \hat{b}]^T$ and corresponding variance matrix $Q_{\hat{x}}$. This result is then input for the second step, where the BLUE of x for the constrained model is obtained as the BLUE of $E\{\underline{\hat{x}}\}$ giving a fixed solution of the unknowns. Hence, the unknowns of the constrained model are determined by [6]:

$$\underline{\check{b}} = \underline{\hat{b}} - Q_{\hat{b}\hat{a}}Q_{\hat{a}}^{-1}(\hat{a} - \check{a}), \quad Q_{\check{b}} = Q_{\hat{b}} - Q_{\hat{b}\hat{a}}Q_{\hat{a}}^{-1}Q_{\hat{a}\hat{b}} \ . \tag{2}$$

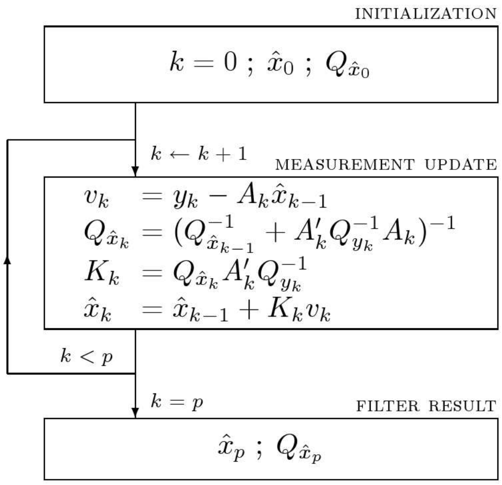


Figure 2: Recursive estimation procedure corresponding with the linear partitioned model Eq.(3)

In a practical application of ILSQ to PS-InSAR the upper described procedure is solved by means of the Least-square AMbiguity Decorrelation Adjustment method (LAMBDA), [6]. For more details on LAMBDA method and its application to PSI see, [6, 2].

2.2 Recursive ILSQ for PSI

The presented mathematical ILSQ for PSI framework serves as a basis for recursive PSI processing. The main goal of recursive PSI is to estimate the parameters of interest recursively from the observed data - double difference interferometric phase observations. The starting point in the analysis is a partitioned model, formed by the solution of the estimates from previous epochs and the observation equations of the new observations:

$$E\{\begin{bmatrix} \underline{\check{x}}_{i|k} \\ \underline{y}_k \end{bmatrix}\} = \begin{bmatrix} I & 0 \\ A_k & -2\pi \end{bmatrix} x_{k|k}; \quad D\{\begin{bmatrix} \underline{\check{x}}_{i|k} \\ \underline{y}_k \end{bmatrix}\} = \begin{bmatrix} Q_{\check{x}_{i|k}} & 0 \\ 0 & Q_k \end{bmatrix}, \tag{3}$$

where parameters related to epoch i prior to new epoch k are $\underline{\check{x}}_{i|k}$ representing estimates of displacement parameters and residual height w.r.t. the reference surface (ellipsoid or a-priori DEM), with corresponding variance-covariance matrix $Q_{\check{x}_{i|k}}$. Parameters related to the new measurement $\underline{y}_k$ are: the ambiguity of the phase observation, a_k, and f_t and $f_{B_{perp}}$ that stand for temporal baseline and height-to-phase conversion factor, [7], while σ_{y_k} stands for standard deviation of the new phase measurement, [8].

2.3 Multi-Modal Adaptive Estimation (MMAE)

There are many different types and implementations of Multi-Modal Adaptive Estimators (MMAE). The MMAE used here follows [4]. A block diagram example of the MMAE filter algorithm is shown in Fig. 3.

An MMAE is a bank or set of a finite number of individual filters, each based on a particular assumption about the system of interest. Each filter within MMAE is called an “elemental filter”. Each elemental filter generates its own estimate of parameters, based not only on the measurements but also on additional assumptions. In this particular case, the assumptions are on different ambiguity vectors. The residual vector and its covariance, Fig. 2, are measures of how well the system modeled in the elemental filter matches the true system. Therefore, if each elemental filter is given the same measurements, then by monitoring residuals it is possible to determine which elemental filter’s model best matches the “true” system.

2.3.1 Computation of conditional probabilities

The residuals v_k are used to calculate the conditional probability of each filter being the correct one. Equation (4) shows how to calculate an individual filter’s conditional probability for time sample t_i and elemental filter (EF) k, [9]:

$$p_{\mathrm{EF}_k}(t_i) = \frac{f\left(\phi_i \mid \mathrm{EF}_k, \Phi_{t_{i-1}}\right) p_k(t_{i-1})}{\sum_{j=1}^{k} f\left(\phi_i \mid \mathrm{EF}_j, \Phi_{t_{i-1}}\right) p_j(t_{i-1})} \tag{4}$$

where the conditional densities for k–th elemental filter can be calculated as:

$$f\left(\phi_i \mid \mathrm{EF}_k, \Phi_{t_{i-1}}\right) = \frac{1}{(2\pi)^{m/2}\sqrt{\mid Q_{\upsilon_k(t_i} \mid}} \exp\{\frac{1}{2}\upsilon_k(t_i)^T Q_{\upsilon_k}(t_i)^{-1}\upsilon_k(t_i)\} \tag{5}$$

where the equation for the recursive update of the lsq-residual variance covariance matrix, adopting the notation of Sec. 2.2, reads:

$$Q_{\upsilon_k}(t_i) = (Q_k + A_k Q_{\check{x}_k}(t_{i-1})A_k^T) \tag{6}$$

In these equations, m is the number of phase measurements, which does not have to be directly related to number of stacked interferograms since the updates can be performed in sets rather then individually. Notice in Eq. (4) that if a filter's conditional probability becomes zero it will remain zero all the time. In order to circumvent this problem a lower probability bound ϵ (generally $0.001 \leq \epsilon \leq 0.01$) is set for each elemental filter. If the calculated probability is less then a lower bound, the filter probability is set to the lower bound.

2.4 Multiple-hypotheses testing: evaluating networks

The purpose of the multiple-hypothesis testing algorithm, [9], is the classification of the conditional probabilities of elemental filters, and hence, the evaluation of the performance of different models that are implemented in the MMAE bank of filters. In the case of PSI, this model difference is represented by the different ambiguity vectors, that are classified in this way. The classification procedure is realized through a decision table, that is populated with conditional probabilities and organized as a function of different hypotheses. The decision table is evaluated by a Maximum A-Posteriori probability criterion (MAP), [9], that is based on a-posteriori probabilities which are recursively computed in each elemental filter. In other words, based on the observation $\phi(t_i)$, the MAP decision rule involves choosing the hypothesis H_i that maximizes the probability of H_i given the double difference phase $\phi(t_i)$, i.e., after the observation has been made. In this way the belief network is designed, [5], which is the performance network of different ambiguity vectors in time. The computation of every node's belief, i.e., its conditional probability, given the evidence that has been observed so far is realized in MMAE filter bank and evaluated by the hypotheses testing algorithm, see Fig. 3.

3 RECURSIVE PSI: ALGORITHM IMPLEMENTATION

This section outlines the algorithm implementation and gives more details on some implementational aspects of the algorithm. The overall design of the current algorithm is shown in Fig. 3. Block #1 is the top level "floating–ambiguity" filter, which treats double–difference phase ambiguities as non-integer, floating point numbers. The ambiguities from this filter are then used to generate candidate ambiguity sets in Block #2. Each of the elemental filters in the multi-modal filtering algorithm is based on the specific one among the candidate ambiguity sets, Block

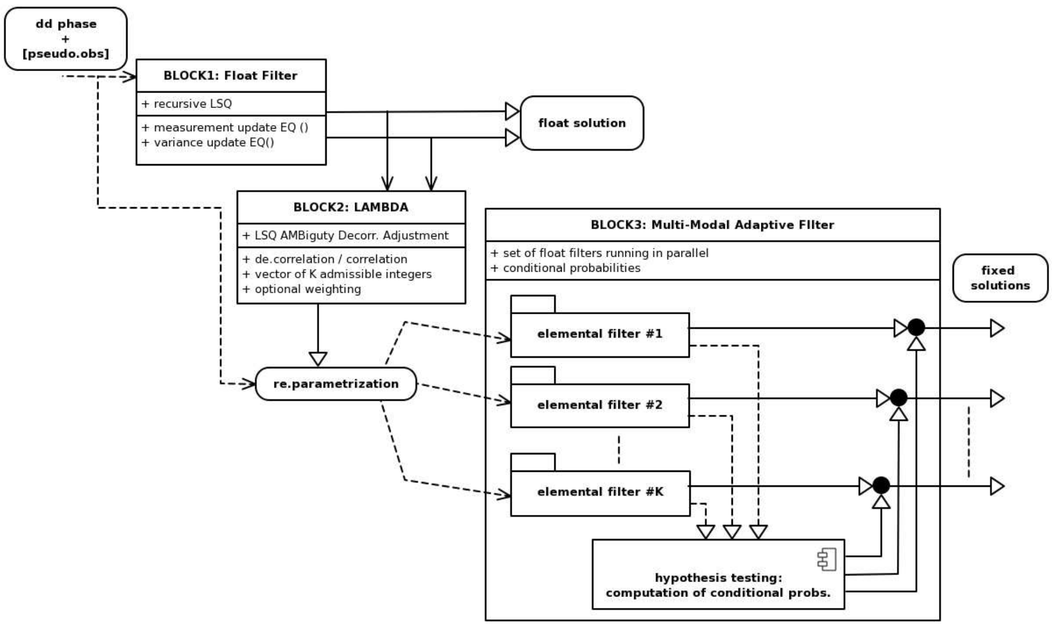


Figure 3: Block diagram: Multi-Modal Adaptive Estimation algorithm. Three structure blocks can be identified: (1) Floating point filter, (2) Candidate ambiguity set generation, (3) MMAE bank of filters.

#3. Residuals generated by each elemental filter are used to calculate and evaluate through hypotheses testing the conditional probability of that elemental filter's estimate being correct.

Floating point filter: The floating point filter presented in Fig. 3 follows from a straightforward recursive estimator explained in Sec. 2.2.

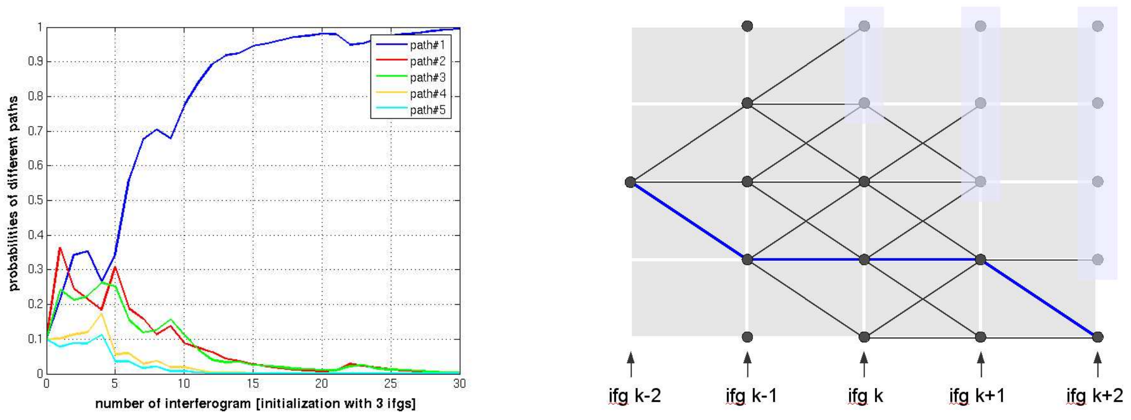
Candidate ambiguity set generation: For this algorithm the important outputs of the floating point filter are the double difference phase ambiguity vectors and their associated covariances. These ambiguity estimates are transformed using the LAMBDA (decorrelation) technique. Note that in the current implementation, there is no optimization in the generation of the candidate ambiguity. During initialization, ambiguity estimates from floating filter are used to generate candidate ambiguity sets after only one update cycle, i.e., estimation starts with a minimum number of three interferograms. This approach is clearly not optimized, and the algorithm would benefit from a more advanced procedure; such as one that would keep some of the older and more promising ambiguity sets and allow floating filter more time to converge.

Conditional probability generation: The conditional probability generation is part of Block #2 and it is based on Eqs. (4),(5). If designed correctly, the elemental filter representing the true system (i.e., having the correct ambiguity vector) will have zero-mean white Gaussian-residuals, with values consistently more in consonance with those computed. It is important to stress out that the success of the whole approach depends on the conditional probability routine being able to distinguish between a correct and an incorrect solution.

Filter pruning through hypotheses testing: An important aspect of the overall algorithm that is not depicted in Fig. 3 is the concept of filter pruning. In order to allow a solution to eventually reach the true fixed point, one of the elemental filters must "absorb" the entire probability weight. However, Eq. (4) shows that this is not possible unless there is only one elemental filter, and hence, only one ambiguity vector in the system. Therefore, a logic was added to the algorithm that would remove an ambiguity vector if the conditional probability of that path remained at the lower bound for more than a specific number of consecutive sample periods, see Fig. 4(b). The number of 10 sample periods was determined by experimenting on simulated data with different strategies and can be varied.

3.1 RESULTS AND ANALYSIS

The algorithm was evaluated using different simulated test scenarios. Figure 4 shows an example of one test case, where ambiguities of the sequantilly stacked interferometric sequence are resolved recursively. It depicts the evolvment of the probability density functions of different ambiguity vectors in time. More specifically from Fig. 4(a) it can be seen that the probability of the incorrect solution could be the highest one, as a consequence of the small number of images. However, if more data is stacked to the system, the probabilities are recursively updated and the system adapts and corrects the probabilities. Finally, the correct option is identified. Note that if correctly implemented, there is no need for smoothing after the correct ambiguity set is determined, since the correct solution is already computed and optionally stored.



(a) Probabilities of different ambiguity vectors as a function of data stacking.

(b) Belief network: The classification of unwrapping tree and branches.

Figure 4: Implementation of recursive ambiguity filter: "pathfinder". Note that the correct ambiguity vector is option 1.

It is obvious that the presented adaptive filter constantly searches for the changes in the system by monitoring the bias in recursively computed residuals. Figure 4(b), depicts the concept of the belief network and filter pruning through the hypotheses testing. The network serves as a "pathfinder", where using the child/parent concept the performance of different ambiguities is evaluated and on this basis the "unwrapping tree" is constructed. Moreover, the tree branches are weighted by hypothesis testing and any branch that does not satisfy the pre–requisite criteria on probability is eliminated. This concept serves as a protection from the false rejection as well as the protection from a false ambiguity acceptance.

Each of the cases demonstrate that the algorithm converges to the correct fixed-integer solution, as long as the correct ambiguity set is available within one of the elemental filters. For a linear deformation model, the presented algorithm converges very quickly. As expected, the convergence time increases in the case of a non–linear deformation. However, due to the assumption of "local" linearity on the deformation signal between time updates, the phase unwrapping is performed correctly.

4 CONCLUDING REMARKS

The main argument for using a recursive processing is the need for parameter estimates in near real-time, and that there is no need to store past measurements for the purpose of computing the current estimates. However, even if the full stack of images were available and batch processing was to be more efficient, the recursive estimation would still have the advantage, as it is easier to analyze the impact of the observation time span and that in the stage of the design of the deformation model ad–hoc adoptions could be made. Intermediate estimates of parameters can be determined as well as the final ones. As mentioned, when a new sensor has been launched, an analysis can be started first after a sufficient number of images has been generated, which is a considerable time span. For these reasons, estimation and testing procedures are developed for recursive PSI, and more specifically for the recursive unwrapping algorithm.

The initial results on simulated data are very encouraging and areas of future work are expected to improve the performance and robustness of the algorithm. Apart from the development and full implementation and performance testing on the real data, several future development efforts can be listed. At the same time, these can be numbered as the main drawbacks of this approach. From the algorithmical point of view, one of the future developments would be the optimization and a better control of the almost exponential growth of the number of options in the filter bank through a better filter's interaction and switching. Furthermore, the algorithm is "embarrassingly" parallel nature and as such, it can be partitioned into multiple streams of execution and efficiently parallelized. From the applicational perspective, the detection of PS candidates with limited set of data and a design of appropriate covariance models for atmosphere would be the directions for further improvements.

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