Dynamic Persistent Scatterers Interferometry

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***Abstract*— This paper presents the concept of Dynamic Persistent Scatterers Interferometry (PSI) processing, which enables the sequential estimation of parameters. The method is based on the Integer Least Squares (ILSQ) PSI concept and makes use of the estimation vector and corresponding variance-covariance matrix of the initial estimation epoch. In addition, the concept of multi–modal adaptive estimation and testing is applied. The algorithm systematically adds a new acquisition or set of acquisitions to an existing stack, updates the solution of the previous run, and analyzes whether the behavior of the (pre-) selected points fits the expected one.**

I. Introduction

Persistent Scatterer Interferometry (PSI) technique has been proved to be an efficient algorithm for the joint estimation of topographic and displacement parameters from a number of interferometric combinations, [1], [2]. However, successful application usually relies on processing all available images at once, i.e., in batch. Therefore, in order to incorporate a newly available acquisition into the estimation, and consequently update the estimates, the whole processing has to be re-performed. Such an approach consequently leads to a cumulative increase in total processing time, it limits the application to those areas where only a sufficient number of images is available, and it reduces the potential application of the method for near-real-time deformation monitoring.

All current multi-image processing techniques, can be described as a hybrid parameter estimation problem; where the estimation quantity has both continuous (e.g., deformation, height) and discrete components (e.g., ambiguity set(s)). Conventional solutions to these estimation problems follow the strategy that can be characterized as "estimation after decision". An algorithm first decides on the best discrete component (i.e., ambiguity vector), and then estimates the continuous component.

More specifically, most PSI algorithms simply "wait" until enough information for the estimation has been obtained, and then derive solution by statistically comparing possible ones, until the correct ambiguity set becomes evident. The disadvantage of this "off-line" approach is, that it ignores the possibility of attaining a solution before the ambiguities are declared "fixed".

In this paper, recent developments on applicability and validation of a new operational methodology for dynamic/recursive PSI are outlined. The concept of dynamic PSI enables the sequential estimation of the parameters of interest. The developed methodology systematically adds a new acquisition, or set of acquisitions, to the existing stack, updates the solution of the previous run, and analyzes whether the behavior of the (pre)selected points fits the expected one. The algorithm builds on concepts of dynamic data processing, [3], integer least squares, [4], and Multi-Modal Adaptive Estimation principles, [5]. Within the presented framework, the ambiguity resolution is treated as a multiple-hypotheses classification problem, [6].

The paper is organized as follows. Section 2 provides background to the relevant mathematical model of PSI; as well as a summary, of conceptual problems and extensions to the mathematical model needed to accommodate for dynamic PSI. Section 3 reviews principles of proposed Dynamic PSI algorithm. Section 4 gives an evaluation overview on real data, as well as implementation aspects. Finally, in Section 5 conclusions and comments on future work are given.

II. Mathematical model of PSI

The general mathematical model for Integer Least Squares PSI approach in Gauss-Markov formulation has the following form:

$$E\{\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix}\} = \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}, \tag{1}$$

$$D\{\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix}\} = \begin{bmatrix} Q_{y_1} & 0 \\ 0 & Q_{y_2} \end{bmatrix} \tag{2}$$

where: $\underline{y}_1$ stands for double-difference phase observations, $\underline{y}_2$ for pseudo observations, $A_{1,2}$ and $B_{1,2}$ are design matrices, a ambiguities ($a \in \mathbb{Z}$), and b unknown parameters of interest ($b \in \mathbb{R}$). To simplify notation $A_{1,2}$ are denoted as A and $B_{1,2}$ as B. Note that in the formulated model, the pseudo-observables are required to solve the model rank-deficiency.

This constrained model is solved by means of Best Linear Unbiased Estimation (BLUE) in two steps, [3]. First, the estimate from the unconstrained linear model,considering $a \in \mathbb{R}$, is obtained, resulting with a float solution $\hat{\underline{x}} = [\hat{a}, \hat{b}]^T$ and corresponding variance matrix $Q_{\hat{x}}$. This result is then input for the second step, where the BLUE of x for constrained model is obtained as the BLUE of $E\{\hat{\underline{x}}\}$ giving a "fixed" solution of the unknowns. Hence, the unknowns of the constrained model are determined by, [3]:

$$\check{\underline{b}} = \hat{\underline{b}} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} (\hat{a} - \check{a}) \tag{3}$$

with corresponding variance matrix:

$$Q_{\check{b}} = Q_{\hat{b}} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} Q_{\hat{a}\hat{b}} \ . \tag{4}$$

In a practical application of ILSQ-PSI the upper described procedure is solved in a three-step procedure, where an extra step is introduced to resolve the ambiguities. This addition is related to an optimization of the ambiguity search spaces and it is realized through the LAMBDA algorithm, [3]. This optimization results in a fixed solution for the ambiguities $\check{a}$. The fixed ambiguities are then used to obtain the solution of the vector of unknown parameters $\check{b}$, by means of Eqs. (3) and (4)

A. *Partitioned model of PSI*

The main goal of recursive algorithms is to estimate the parameters of interest sequentially from the observed data. The starting point in the analysis is partitioning of Eq. (1) model. The partitioned model is formed of the solution of the estimates from previous epochs and observation equations of new observations:

$$E\{\begin{bmatrix}\check{\underline{x}}_{i|k} \\ \underline{y}_k\end{bmatrix}\} = \begin{bmatrix} I & 0 \\ A_k & -2\pi \end{bmatrix} x_{k|k};\ D\{\begin{bmatrix}\check{\underline{x}}_{i|k} \\ \underline{y}_k\end{bmatrix}\} \quad = \begin{bmatrix} Q_{\check{x}_{i|k}} & 0 \\ 0 & Q_k \end{bmatrix}, \tag{5}$$

where parameters related to epoch i prior to new epoch k are $\check{\underline{x}}_{i|k}$ representing estimates of displacement parameters and residual height, with corresponding variance-covariance matrix $Q_{\check{x}_{i|k}}$. Parameters related to the new measurement $\underline{y}_k$ are: the ambiguity a_k, and f_t, $f_{B_\perp}$ that represent the temporal baseline and height-to-phase conversion factors respectively, while σ_{y_k} represents the for standard deviation of the new phase measurement. The mathematical framework in which recursive updates are performed is given by Figure 1.

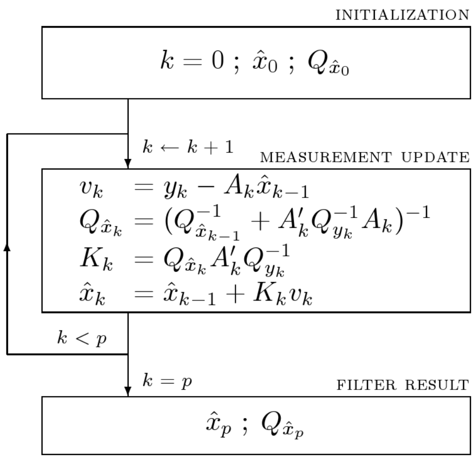


Fig. 1. One cycle of the recursive estimation procedure, corresponding with the partitioned model Eq.(5)

B. *Influence of pseudo observations*

In case of PSI the pseudo-observables and the accompanying variance matrix cause a bias in the solution. This is the result of the uniqueness of the solution of the system of equations Eq. (1), that is, the lack of redundancy. Due to this uniqueness the variance matrix Q_y used does not influence the estimated parameters $\hat{a}, \hat{b}$. Hence, any value can be adopted for the variance of the pseudo-observables (Q_{y_2}) without influencing the estimates. However, the choice of Q_{y_2} does influence the variance matrices of the estimates $Q_{\hat{a}}, Q_{\hat{b}\hat{a}}$. This shows that there is no basic relationship between the estimates and the accompanying variance matrices. Results obtained with Eq. (3) are therefore unreliable and, hence, a straightforward application of the recursive model not possible. In the current implementation this problem is solved by means of Variance Component Estimation (VCE), [7].

III. Concept of dynamic PSI estimation

As all other ambiguity resolution algorithms, the dynamic PSI algorithm is based on monitoring the measurement residuals by statistically comparing possible solutions, until the correct ambiguity set becomes evident. In the case of a dynamic approach, these statistics is gathered and evaluated sequentially, that is, without algorithmic need to save previous estimates. However, in a case of sequential estimation, the rank deficiency usually a minimum number of images is necessary, see Figure 2. Therefore, the conventional recursive approach, see Figure 1, for the successful application needs to be extended.

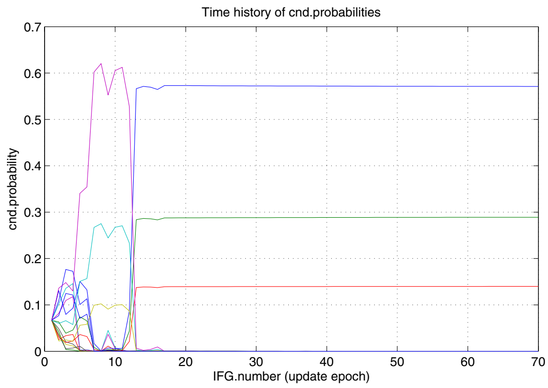
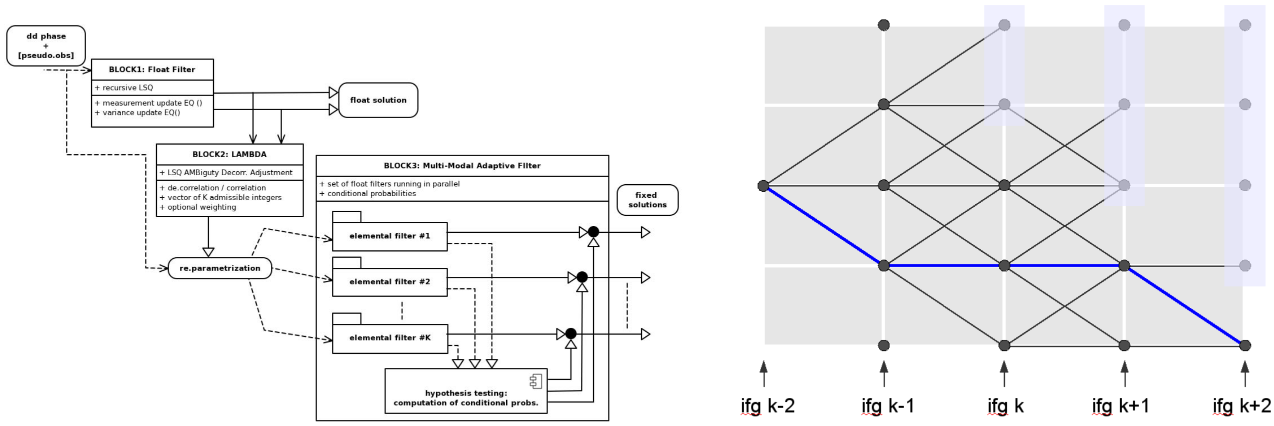


Fig. 2. Recursively computed time history of conditional probabilities (see Eq. (6)) of different ambiguity resolution vectors of one arc. Each conditional probability, hence ambiguity vector, is visualized in different color. Note that the correct solution, given the data, is the one showing the highest probability, and that the correct solution cannot be clearly indicated with limited set of images. In this example, after 14 updates the correct solution is indicated, thus, fixing on a certain ambiguity vector with less then 14 images in the system would result with incorrect estimates.

The basic idea of the algorithm is based on the construction of a "belief network", see Figure 3(b), based on the previous estimates, [5]. This idea is realized through a finite set of estimators, each of which hypothesizes a different ambiguity candidate set. The residual vector and its covariance, computed



(a) Dynamic PSI algorithm: algorithm building blocks for one branch of belief network. In each recursive update estimates and corresponding vc matrix of parameters of interest are computed/updated w.r.t. previous estimates. The evaluation of different ambiguity vectors and computation of corresponding conditional probabilities used for their validation is computed in hypothesis testing block.

(b) Concept of "belief network", each of the ambiguity candidate vectors is recursively evaluated through conditional probabilities, see Figure 2. Tree branches showing the least likelihood of being correct ones are eliminated from further evaluation. The black dots indicate ambiguities, while lines connecting them ambiguity vectors. Shaded block are recursively rejected ambiguity vectors.

Fig. 3. Flowchart of Dynamic PSI algorithm.

as the difference between the measurement, y_k, and the filter's prediction of the measurements before they arrive, are then evaluated by the hypotheses testing algorithm. The output of the testing gives a relative indication of how close each ambiguity set, used in various filters, is to the "true" ambiguity model.

A. Multi-Modal Adaptive Estimation (MMAE)

The belief network, used for the evaluation of the ambiguity vectors is constructed by means of the Multi-Modal Adaptive Estimation algorithm, [5]. MMAE is a bank of individual estimation filters each based on a particular assumption about the system of interest. Each filter generates its own optimal estimate of the parameters of interest, based not only on the system measurements, but also on its own assumptions of the system. In the case of dynamic PSI the system assumptions relate to different ambiguity vectors. The residual vector and its covariance, see Figure 1, are measures of how well the ambiguity vector in the estimation filter matches the true one.

1) Computation of conditional probabilities: The residuals υ_k, see Fig. 1, are used to calculate the conditional probabilities of each estimation filter, thus the ambiguity vector assigned to that particular filter, being the correct one. Equation (6) shows how to calculate the conditional probability for one filter for time sample t_i and elemental filter k:

$$p_{\mathrm{EF}_k}(t_i) = \frac{f\left(\phi_i \mid \mathrm{EF}_k, \Phi_{t_{i-1}}\right) p_k(t_{i-1})}{\sum_{j=1}^{k} f\left(\phi_i \mid \mathrm{EF}_j, \Phi_{t_{i-1}}\right) p_j(t_{i-1})} \tag{6}$$

where the conditional densities for k–th elemental filter is calculated as:

$$f\left(\phi_i \mid \mathrm{EF}_k, \Phi_{t_{i-1}}\right) = \frac{\exp\{\frac{1}{2}\upsilon_k(t_i)^T Q_{\upsilon_k}(t_i)^{-1}\upsilon_k(t_i)\}}{(2\pi)^{m/2}\sqrt{\mid Q_{\upsilon_k(t_i)} \mid}} \tag{7}$$

where the equation for the recursive update of the LSQ-residual variance covariance matrix is calculated as:

$$Q_{\upsilon_k}(t_i) = (Q_k + A_k Q_{\hat{x}_k}(t_{i-1}) A_k^T) \tag{8}$$

where m is the number of phase measurements in the update. An example of recursively computed conditional probabilities and their time history is given in Figure 2.

B. Multiple-hypotheses testing: evaluation and rejecting of ambiguity candidates

An important aspect of the overall algorithm that is the concept of elemental filter rejection. In order to allow a solution to eventually reach the correct solution, one of the estimation filters must "absorb" the entire probability weight. However, Eq. (6) shows that this is not possible unless there is only one elemental filter, and hence, only one ambiguity vector in the system. Therefore, a logic was added to the algorithm that would remove an ambiguity vector if the conditional probability of that path remained at the lower bound for more than a specific number of consecutive sample periods. The recommended number, in which rejection / merging should be performed, is 5-10 sample periods.

The principles of the Dynamic PSI algorithm are depicted in Figure 3, where a flowchart of the algorithm is given as well as an example of a "belief network".

Dynamic PSI algorithm constantly searches for the changes in the system by monitoring the bias in recursively computed residuals. The "belief network" serves as a "path-finder", where using the child/parent concept of genetic algorithms, the performance of different ambiguities is evaluated and on this basis the "unwrapping tree" is constructed. Moreover, the tree branches are weighted by hypothesis testing and any branch that does not satisfy the pre–requisite criteria on probability is rejected. This concept serves as a protection from the false

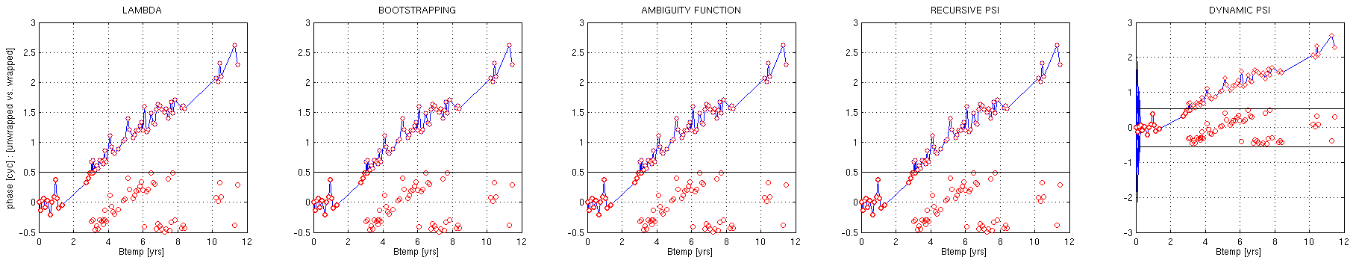


Fig. 4. Sample are 1. Temporal unwrapping, estimation per-arc: All algorithms retrieved same estimate. From left to right, the graphs depict the results from various unwrapping algorithms.

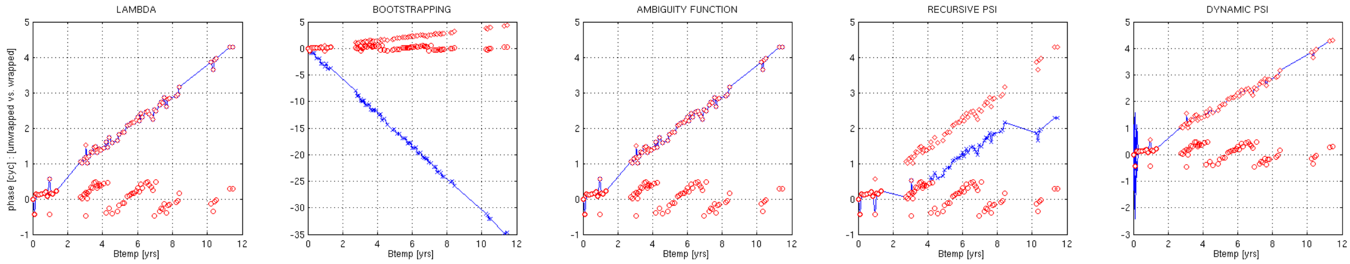


Fig. 5. Sample arc 2. Due to the temporal sampling, estimation per-arc: Due to temporal sampling of data, both bootstrapping and recursive ILSQ, estimated incorrect results.

rejection as well as the protection from a false ambiguity acceptance.

IV. Results and Algorithm Analysis

The performance Dynamic PSI algorithm is evaluated first on a single-arc basis, through cross-comparison with different unwrapping/PSI estimation concepts, i.e., ILSQ, bootstrapping, ambiguity function, recursive model based on ILSQ, and dynamic PSI estimators are cross-compared.

For validation real life PSI estimates are used,related to 12 years of ERS data. The results of standard processing using a linear deformation model for tow independent arcs are shown in Figures 4 and 5.

For cross-comparison, batch-processed ILSQ-PSI estimates are used as a reference. Two independent arcs are analyzed. The strategy for evaluation was that estimates of ILSQ, bootstrapping and the ambiguity function, are obtained in batch, i.e., processing all available images at once. For the recursive ILSQ model, an initialization with 10 interferograms was performed, while updates were with 2 acquisition. The Dynamic PSI results are obtained with 3 interferograms for the initialization, and with consecutive updates of 1 acquisition.

Results indicate that the Dynamic PSI approach, even though initialization was performed with small number of images would find the "correct" solution (i.e., the ILSQ-PSI) just after a few updates. The flexibility of the Dynamic PSI algorithm is demonstrated in Figure 5, where both the bootstrapping and the recursive model estimation failed to estimate the correct ambiguity vector.

V. Concluding Remarks

The initial results on real data are encouraging and support studies to improve the performance and robustness of the algorithm. Apart from the development and performance testing of the dynamic 1D+2D unwrapping algorithm, developments would be the extension of validation strategies to sequentially estimated parameters, as well as study on the influence of geometrical characteristics of the stack on Dynamic PSI.

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