PPE 11 Principle of linearized stability.

Consider

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Suppose that a, b, c and d are all positive.

Exercise 1.
(i). Show that M has two different eigenvalues λ_1 and λ_2.

(ii). Prove that you can choose the eigenvalue λ_1 in such a way that

$$|\lambda_2| \leq \lambda_1$$

Let

$$P = \begin{pmatrix} -\frac{1}{2} & 1 \\ \frac{1}{2} & a \end{pmatrix}$$

Let T denote the trace of P and D the determinant.

Exercise 2.
(i). Sketch in the (T, D) plane the triangle formed by $D = T - 1$, $D = -T - 1$ and $D = 1$.

(ii). Sketch in the same picture the actual position of (T, D) for P, depending on a.

(iii). For which values of a is the steady state $x = 0$ a stable one for the linear recursion

$$x(n+1) = Px(n)$$