IDEALIZATION PRECISION OF POINT SCATTERERS FOR DEFORMATION MODELING

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ABSTRACT

Although PS-InSAR [1] deformation measurements may be very precise in terms of dispersion, this does not automatically imply a reliable estimation of the signal of interest. It is necessary to determine the *idealization precision* for deformation modeling: how well do the Persistent Scatterer (PS) measurements represent the displacements caused by the signal of interest?
In the context of separation of displacements due to a superposition of deformation regimes [2], this research focuses on the Subsidence Residual modeling (SuRe) concept. SuRe uses an integral geodetic mathematical model, with the differences between measurements and subsidence prognosis as observations. The residuals due to benchmark setting and deviations from the subsidence prognosis are modeled stochastically. The stochastic model parameters are estimated through Variance Component Estimation (VCE) [3]. Using geodetic adjustment and testing techniques combined with VCE, displacements due to benchmark setting and spatially correlated subsidence can be separated.
This study investigates VCE for estimating stochastic model parameters in case of a superposition of deformation regimes, based on simulated residual observations. Furthermore the SuRe concept has been applied in a test area using both real leveling and PS-InSAR data. The results show that subsidence caused by a contaminated deformation regime can be estimated precisely, but that awareness of the idealization precision of the deformation measurements is important for reliable results.

1 IDEALIZATION PRECISION FOR DEFORMATION MODELING

The availability of precise geodetic measurements from techniques such as PS-InSAR and leveling is not necessarily sufficient for a precise and reliable estimation of the deformation signal of interest. One needs to know the physical relation between measurement and foundation layer(s), combined with the possible presence and magnitude of other deformation phenomena: gas, salt or water extraction, polder drainage, natural compaction, isostacy or tectonic effects. In other words, knowledge about the idealization precision for deformation modeling is required: how well do the measurements represent the displacements caused by the signal of interest?
In classical geodesy, the idealization precision gives an indication of the identification precision of a point in the terrain. Points with a high idealization precision, like the corner of a house, can be sharply identified. Points like the middle of a canal have a worse idealization precision. This idealization precision is taken into account in the stochastic model, to enable a correct estimation of the point coordinates. The same concept is applicable to deformation parameter estimation from geodetic measurements. For example: if the signal of interest, subsidence due to gas extraction, is contaminated by shallow subsurface deformation of a similar magnitude, it has a low idealization precision. However, by (stochastically) modeling the idealization precision, it should still be possible to reliably estimate gas extraction subsidence with a high precision.
Although idealization precision in deformation modeling plays a role in each geodetic measurement technique, it is more prominent in PS-InSAR than in traditional techniques like leveling or GPS, due to the physical properties of the measurement points. Leveling uses well defined benchmarks, which may have been established in a building founded on a stable subsurface layer. In case of SAR reflections, the physical measurement point can be less sharply identified. It may be even harder to determine to which foundation layer(s) they refer to due to possible multi-bounce reflections. Furthermore, in urban areas where the PS density is usually high, numerous spatial and temporal changes may be going on, leading to variations in surface burden and (delayed) subsurface compaction.

In a previous study [2] it was shown that PS-InSAR deformation measurements can have a high local variability which hampers the recognition of the signal of interest as it is contaminated by other spatio-temporal effects. This superposition of different deformation regimes occurs frequently in areas with soft soils. Based on a cross-validation technique using Ordinary Kriging, PS's were selected that fitted the spatio-temporal properties of the signal of interest. It is preferable however to use all deformation measurements to benefit from the highest possible redundancy for parameter estimation. To enable this, the measured displacement should be split up into components due to different deformation causes.

2 CHARACTERIZATION OF DEFORMATION REGIMES

2.1 Deformation regimes

A measured displacement may be decomposed into displacements caused by different deformation regimes, as shown in Fig. 1. Displacements caused by different deformation regimes usually have different spatio-temporal characteristics. Deformation regimes can be generally classified in the following way:

- structural instabilities (foundation, benchmark setting, pile friction),
- shallow mass displacements (groundwater level variations, polder drainage, natural compaction of shallow layers),
- deep mass displacements (gas, oil and salt extraction, tectonics, isostacy).

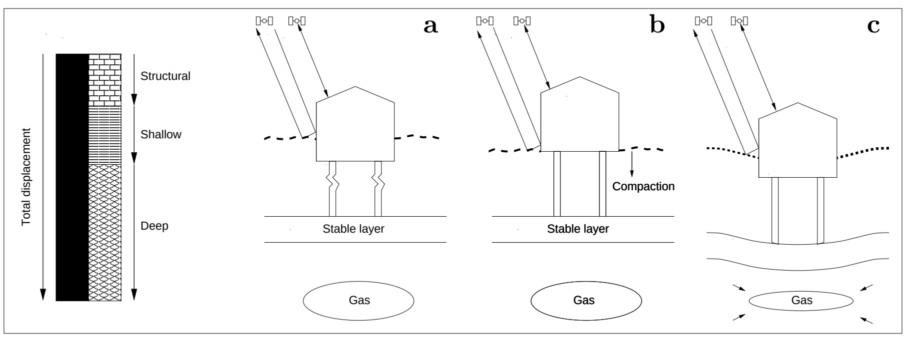

Figure 1: Example of a measured displacement decomposition due to different deformation causes: (a) foundation, (b) compaction, (c) gas extraction.

2.2 Deformation regime parameters

The behavior of deformation regimes can be very complex. The number of parameters needed to characterize them in a functional model may exceed the number of observations and lead to an unsolvable system. Another more generic approach is modeling differences between measurements and the prognosed deformation stochastically, based on the different spatio-temporal behavior of these residuals for different deformation regimes. For each deformation regime, a covariance function has to be defined containing the stochastic model parameters that describe the spatio-temporal behavior of the residuals. This covariance function is used to construct the variance-covariance matrix (vcm) of the 'residual' observations. An example of such a function for both temporally and spatially correlated subsidence reads:

$$C\{\underline{\delta z}_i^t, \underline{\delta z}_j^u\} = \frac{1}{2}\sigma^2(|t|^{2p} - |t-u|^{2p} + |u|^{2p})e^{-(l_{ij}/L)^2} \tag{1}$$

where

$C\{\underline{\delta z}_i^t, \underline{\delta z}_j^u\}$	covariance between subsidence residual δz at point i on time t and δz at point j on time u,
σ^2	variance,
p	temporal power,
L	spatial correlation length,
l_{ij}	distance between point i and j.

This covariance function may lead to a semi-positive definite vcm, but when adding the positive definitive vcm due to measurement noise to it, the total vcm will be positive definite. The spatio-temporal behavior of the subsidence residuals are parametrized by σ^2, L and p. Other deformation regimes with different stochastic properties (spatial and temporal correlation length, isotropic or anisotropic) will have different stochastic model parameters and different covariance functions.

2.3 Related techniques

Modeling deformation regimes stochastically using covariance functions to construct a vcm for the deformation observations is strongly related to techniques like factorial kriging and principal component analysis [4]. In case of factorial kriging, nested variograms are constructed to map each spatial component separately. The total variogram is a superposition of variograms with different correlation lengths. To use factorial kriging, knowledge of the different correlation lengths of the 'deformation regimes' is necessary.

In Principal Component Analysis (PCA) the eigenvalues and eigenvectors of the vcm specify the magnitude and direction of the stochastic processes covered by the vcm. A disadvantage of PCA is that physical interpretation of the eigenvalues and eigenvectors is often not straightforward.

Recognition of the presence of time-correlated noise besides white noise in geodetic time series is described in [5]. Two techniques to estimate the noise parameters are evaluated: spectral estimation and the so called 'M estimates'. Spectral estimation has the disadvantage that regular sampling is required and estimates appeared to be not very accurate. 'M estimates' is a maximum likelihood technique, maximizing a probability function by adjusting the magnitudes of the noise types.

3 ESTIMABILITY OF DEFORMATION REGIMES

3.1 Geodetic mathematical model

To estimate unknown parameters from observations including a quality description, the Delft adjustment and testing theory [6; 7] constructs an integral mathematical model consisting of both a functional and a stochastic part:

$$E\{\underline{y}\} = Ax \qquad D\{\underline{y}\} = Q_y \tag{2}$$

where

$\underline{y}$	stochastic vector of observations,
Q_y	variance-covariance matrix of observations,
A	design matrix,
x	vector of unknown parameters.

The functional model describes the relation between the observations and the unknown parameters; the stochastic model consists of the vcm of the observations. The vcm may not only contain the measurement noise, but also the noise due to different deformation regimes in the observed displacements. Observation errors and errors in the functional model are traced by specifying alternative hypotheses and comparing their test statistics to a critical value [7]. If the test statistic of the Overall Model Test is largest and not equal to its expectation value of 1, a Variance Component Estimation (VCE) of the stochastic model is carried out. The stochastic model parameters are estimated based on the adjustment residuals.

3.2 VCE: separation of benchmark setting and spatially correlated subsidence

Specifying point noise and model noise in the stochastic model has been applied in the Subsidence Modeling (SuMo) concept developed in cooperation between Delft University of Technology and Nederlandse Aardolie Maatschappij B.V. (NAM) [8]:

- point noise is the temporally correlated, but spatially uncorrelated displacement of measurement points. This component is due to individual point characteristics, such as foundation pressure and pile friction, whereas
- model noise is the temporally and spatially correlated difference between the prognosed and the actual subsidence. This component is due to imperfections in the prognosed total subsidence, due to possibly overlapping extractions of gas, water and salt, polder level changes and compaction, isostacy and tectonics.

The SuMo concept has been further developed to Subsidence Residual modeling (SuRe) by Adriaan Houtenbos, using as observations the differences between measurements and the subsidence prognosis based on reservoir information. The measurements are spatial height differences on a certain time. Note that due to the differential character of the measurements, a constant bias could exist that cannot be estimated. Because of this, the estimated subsidence has a relative character. The bias can be calculated by tuning the results with the subsidence prognosis outside SuRe.

The stochastic model covers the non-spatially correlated benchmark behavior and all other accumulated spatially correlated subsidence due to both shallow and deep compaction. Q_y, the vcm of the observations, is constructed as a superposition of measurement, point and model noise, which are described by their own covariance functions. As the measurements are spatial differences, Q_y contains the noise due to differential measurement precision, differential benchmark setting and differential deviations between the prognosis and the actual subsidence.

An improvement in the SuRe concept is the Variance Component Estimation of the stochastic model parameters. To enable this, Q_y is decomposed as a Taylor polynomial with the first derivatives of the vcm to the stochastic model parameters, which are called the variance factors:

$$Q_y = Q_y^0 + \Delta\sigma^2_{obs}(dQ_{obs}/d\sigma^2_{obs}) + \Delta\sigma^2_{stb}(dQ_{stb}/d\sigma^2_{stb}) + \Delta\sigma^2_{mod}(dQ_{mod}/d\sigma^2_{mod}) + \Delta p(dQ_{stb}/dp) + \Delta q(dQ_{mod}/dq) + \Delta L(dQ_{mod}/dL) \quad (3)$$

where

$Q_{obs}, Q_{stb}, Q_{mod}$	partial vcm's covering measurement, point and model noise,
$\sigma^2_{obs}, \sigma^2_{stb}, \sigma^2_{mod}, p, q, L$	stochastic model parameters: measurement, point and model variance; temporal power of point and model noise; spatial correlation length of model noise,
dQ	cofactor matrix containing the derivative of Q_y with respect to the noise parameter.

Note that the stochastic model parameters in the SuRe concept are not absolute factors, as the observations are spatial differences. The model noise covers a spectrum of spatial wavelengths of superposed spatially correlated deformation regimes. It has to be investigated to which extent their parameters can be estimated separately. This is e.g. dependent on the precision of the estimated variance factors (correlation lengths).

3.3 Precision of VCE

Variance Component Estimation is based on the weighted sum of least-squares residuals, estimating the variance factors according to the decomposition of the vcm to physical interpretable variance factors. The precision of the variance factors is dependent on the redundancy in the mathematical model (in case of full rank: number of observations minus number of unknowns) and the expectation value of the variance factor. One can see this clearly in case of only one unknown variance factor for Q_y. The estimator and the precision of the estimate read [3]:

$$\underline{\hat{\sigma}}^2 = \frac{\underline{\hat{e}}^T Q^{-1} \underline{\hat{e}}}{m-n} \quad ; \quad \sigma^2_{\hat{\sigma}^2} = \frac{2\sigma^4}{m-n} \quad (4)$$

where

$\underline{\hat{\sigma}}^2$ variance factor estimator,
Q cofactor matrix of $Q_y = \sigma^2 Q$,
$\underline{\hat{e}}$ vector of least-squares residuals,
$m - n$ redundancy.

Besides the redundancy, the spatial and temporal sampling frequency of the measurements is important. Correlation lengths lower than the distance between measurement points cannot be estimated.
The variance factor estimators are unbiased and have minimum variance in case the observations are normally distributed [3]. Estimated variance factors are not necessarily positive. In case they are negative, the model redundancy may not be sufficient or the decomposition of the vcm may not be adequate.
To verify the estimability of the variance factors, simulations have been carried out. In these simulations, observation noise has been generated based on the measurement noise and the noise due to the presence of one or more deformation regimes. Therefore Q_y was created using a superposition of covariance functions. Using the Cholesky decomposition $Q_y = R^T R$, observation noise was generated as $dy = R^T n$, with n a vector of standard normally distributed variables.
The VCE simulations focus on the precision of the variance factor estimates. Actually, the vector of 'residual' observations is not necessary to make a precision assessment of the variance factor estimates. The design matrix and the decomposition of Q_y into cofactor matrices are sufficient to do this. The ratio between the precision and the estimated value of a variance factor determines its level of significance and subsequently the separability of the deformation regime it describes. The varying elements in the simulations are:

- spatial and temporal sampling frequency (number of measurement points and epochs),
- measurement noise and number of deformation regime parameters.

As most of the cofactor matrices of the deformation regimes are full matrices, VCE requires a lot of processing time. The simulations have therefore been restricted to small networks.

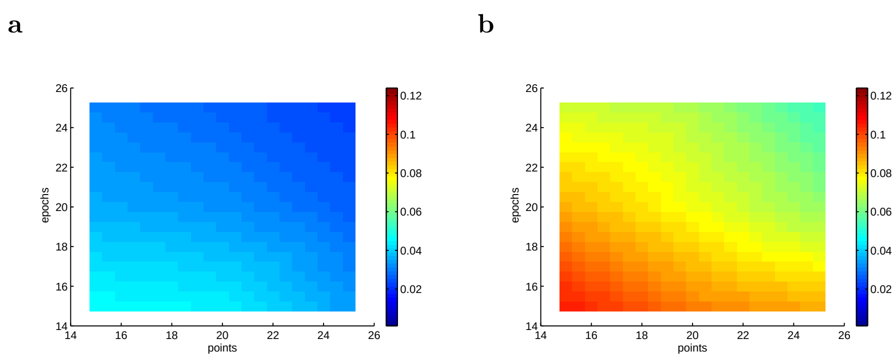

Figure 2: Precision of the variance factor $\hat{L}$ with expectation value 0.25 in case of (a) 4 (measurement and model noise) and (b) 6 (measurement, point and model noise) variance factors to be estimated. The precision is varying with number of measurement points, number of epochs and redundancy.

If the number of variance factors to be estimated increases while the network redundancy stays the same, the precision of the estimated variance factors decreases, as can be deduced from comparing Fig. 2a to Fig. 2b. The higher the spatial and temporal sampling frequency, the higher the redundancy and the higher the level of significance of the variance factor estimates. For real world applications the match with the simulated pattern has to be verified.
The variance factors can only be estimated if the VCE components are independent. If one of the cofactor matrices can be constructed as a linear combination of the other cofactor matrices, the system of equations to be solved will be singular. This occurs for example when two point noise deformation regimes are present with the same temporal behavior, but with different variance. These cannot be separated.
Note that for the usage of VCE for deformation regimes, vcm derivatives are functions of more than one variance factor if the covariance function of a deformation regime contains more than one parameter. For example: to calculate $dQ_{mod}/d\sigma^2_{mod}$, the values for the variance factors q and L are needed as well. An iterative procedure has therefore to be followed the find the optimal estimates for all variance factors. The impact of the iterative

procedure on the variance factor estimates and their precision has to be further studied.

4 SUBSIDENCE ESTIMATION WITH PS-INSAR AND LEVELING

4.1 InSAR and leveling measurement type

Besides a different idealization precision for deformation modeling, PS-InSAR and leveling measurements are information-wise not the same. A leveling measurement $\underline{h}_{ij}^{t}$ is a spatial height difference between points i and j on time t. A PS-InSAR measurement $\underline{d}_{i}^{mt}$ is a temporal interferometric difference between master time m and slave time t, for a certain point i.
Because of different physical properties of the measurement points, benchmark versus reflection, it is unlikely that both measurements can be compared directly. To use leveling and PS-InSAR measurements within an integral model, the different deformation regimes present in both types of measurements have to be taken into account, to allow for an unbiased estimation of the the signal of interest.
The first PS-InSAR observations that bear interpretable information are the double-differences $\underline{d}_{ri}^{mt}$ between master and slave time and spatially with respect to a reference point r [9]. They can be reformulated like leveling type of observations, setting the spatial deformation in the master image to 0:

$$\underline{d}_{ri}^{mt} = \underline{d}_{ri}^{m} - \underline{d}_{ri}^{t} \quad ; \quad \underline{d}_{ri}^{m} = 0 \quad \rightarrow \quad \underline{d}_{ri}^{mt} = -\underline{d}_{ri}^{t} \tag{5}$$

4.2 Functional model and variance factor precision

To solve for the parameters of interest in an integral adjustment and testing procedure, the leveling and PS-InSAR observations have to be combined in the same functional model. The functional relation between observations and unknowns can be written as a model of observation equations or as a model of condition equations [6]. For the precision of the variance factors, the redundancy in the mathematical model is important.
For the PS-InSAR technique, the redundancy in both model formulations can be determined in the following way. If the number of PS's is equal to P, the number of independent spatial differences that can be formed is $(P-1)$. With T epochs or acquisitions available, the number of observations will be $(P-1)(T)$. The spatial differences in the master image are set to 0 and their stochasticity is accounted for in Q_y; they are entered as so called pseudo-observations. The number of unknowns are the point heights at one deformation reference date. As heights can never be deduced from height differences, the reference height of one point is held fixed, which leads to $(P-1)$ number of unknowns. In case the design matrix is of full rank, the redundancy is equal to the number of observations minus the number of unknowns: $(P-1)(T) - (P-1) = (P-1)(T-1)$.
When using double-differences, conditions are formed stating the double-differences minus the prognosed deformation should be equal to 0. Noise types related to the deformation regimes are accounted for in the stochastic model. In this case the redundancy is equal to the number of conditions that can be formulated, so equal to the number of independent double-differences that can be formed. Spatially $(P-1)$ independent differences can be constructed, for each interferogram. From T SAR acquisitions $(T-1)$ independent interferograms can be calculated, which leads to a redundancy of $(P-1)(T-1)$.
As the redundancy in the model of observation equations is equal to the redundancy in the model of condition equations, the variance factors can be determined with an equal precision.

4.3 Subsidence estimation of a deformation regime using InSAR and leveling

As a first indication of the applicability of the Subsidence Residual modeling concept, tracing erroneous observations and applying VCE, a test run has been carried out in a small area of approximately 100 km^2, successively with real leveling and PS-InSAR measurements. In case of PS-InSAR, the spatial and temporal sampling frequency has been varied. PS-InSAR points have been selected randomly, as the goal is using all PS-InSAR observations, independent of the present deformation regimes.
The precision of all PS-InSAR observations have been assumed to be uncorrelated, like the leveling measurements. However it is likely that the measurement noise of PS-InSAR observations is spatially and temporally

correlated due to atmosphere and deformation estimation in the PS-InSAR processing and that it is also dependent on the physical properties of the PS and its surroundings (Signal to Clutter Ratio). Furthermore, the assumptions regarding linear or more complex temporal deformation patterns of the PS's will influence the PS displacement observations. This falls outside the scope of this study, but will be addressed in the future.
The SuRe approach with deformation regimes point noise and model noise has been applied to estimate spatially correlated subsidence which may be due to gas extraction. Table 1 lists the estimated variance factors and their precision for three test runs. Fig. 3 and 4 show respectively the estimated subsidence and its precision.

Type	Leveling	InSAR	InSAR
Observations	716	1710	2082
Unknowns	244	172	348
σ_{obs}	0.83±0.03	1.38±0.03	1.44±0.04
σ_{stb}	0.70±0.05	0.73±0.05	0.81±0.04
σ_{mod}	0.93±0.13	0.99±0.13	0.72±0.09
L	1933±335	2048±277	1530±273
p	0.89±0.02	0.94±0.01	0.95±0.01

Table 1: Estimated variance factors and their precision.

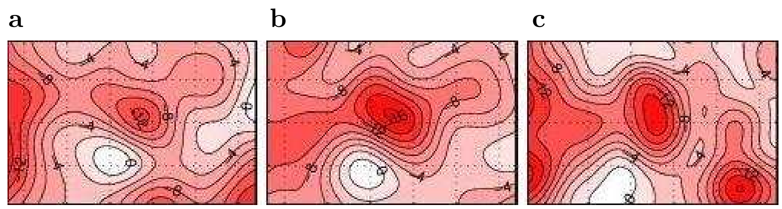

Figure 3: Subsidence estimation (mm) using (a) leveling, (b) PS-InSAR and (c) PS-InSAR with a higher spatial density. PS-InSAR data has been processed by TRE/NPA for NAM. There is a constant bias in each estimation due to the differential character of the observations.

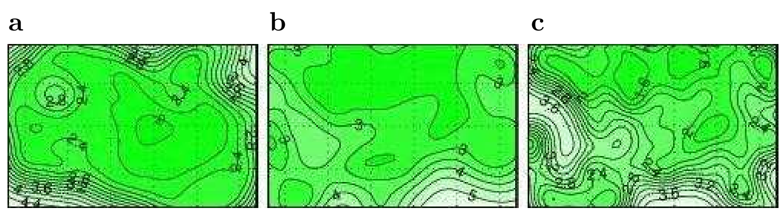

Figure 4: Precision subsidence estimation (1sd, mm) using (a) leveling, (b) PS-InSAR and (c) PS-InSAR with a higher spatial density.

Table 1 shows a lower measurement precision for PS-InSAR than for leveling, while the point noise variance is of an equal magnitude. Due to the high local variability of the PS-InSAR measurements one would expect both a higher measurement and point noise variance for PS-InSAR. However, SuRe also performs a test of the functional model, and PS's with an abnormal velocity have been rejected. The model noise correlation length decreases when the local variation in the estimated subsidence increases. This may be caused by a superposition of spatially correlated deformation regimes. The temporal power is around 1, which means a near linear behavior of the subsidence displacement. For PS-InSAR this can as well be addressed to assumptions during the PS processing.
From Fig. 3 and Fig. 4 it appears that also a contaminated signal of interest can be reliably estimated with a high precision, when taking the displacement due to benchmark setting and subsidence prognosis imperfections into account in the stochastic model, together with VCE and a sufficient spatial and temporal measurement density. Leveling and PS-InSAR generally show the same subsidence pattern. This subsidence pattern is the accumulated subsidence due to all spatially correlated deformation regimes. The subsidence estimation from PS-InSAR shows more local variations, especially when the spatial density of measurement points increases.

This indicates that the residual subsidence component contains displacements due to unmodeled deformation regimes. In future research, it will be investigated if both the functional part, in terms of the subsidence prognosis, and the stochastic part, in terms of deformation regimes with different spatial correlation lengths, can be improved for this test area.

5 CONCLUSIONS

Awareness of the idealization precision for deformation modeling is important for a reliable estimation of the signal of interest. When using all available displacement measurements, they have to be decomposed into contributions caused by the present deformation regimes. In this context, the Subsidence Residual modeling concept has been introduced. SuRe uses the differences between measurements and prognosed deformation as observations and makes a distinction between measurement, benchmark setting and prognosis imperfections in the stochastic model. Stochastic model parameters are estimated using Variance Component Estimation. The precision of the estimated stochastic model parameters, the variance factors, depends on the redundancy and the spatio-temporal measurement density. The more variance factors to be estimated the worse the precision, in case the redundancy stays the same. Applications of SuRe to both real leveling and PS-InSAR data in a test area, shows that also a contaminated signal can be precisely estimated. However, the subsidence residuals indicate the presence of unmodeled deformation regimes. Future research will focus on the superposition of more spatially correlated deformation regimes and the validation of VCE.

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References

[1] A. Ferretti, C. Prati, and F. Rocca, "Permanent scatterers in SAR interferometry," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 39, no. 1, pp. 8–20, Jan. 2001.

[2] V. B. H. Ketelaar and R. F. Hanssen, "Separation of different deformation regimes using INSAR data," in *Third International Workshop on ERS SAR Interferometry, 'FRINGE03', Frascati, Italy, 1-5 Dec 2003*, 2003, p. 6 pp.

[3] P. J. G. Teunissen, *Towards a Least-Squares Framework for Adjusting and Testing of both Functional and Stochastic Models*, ser. Mathematical Geodesy and Positioning series, 1988, vol. 26.

[4] M. van Meirvenne and P. Goovaerts, "Accounting for spatial dependence in the processing of multi-temporal sar images using factorial kriging," *International Journal of Remote Sensing*, vol. 23, no. 2, pp. 371–387, 2002.

[5] J. Langbein and H. Johnson, "Correlated errors in geodetic time series: Implications for time-dependent deformation," *Journal of Geophysical Research*, vol. 102, no. B1, pp. 591–603, Jan. 1997.

[6] P. J. G. Teunissen, *Adjustment theory; an introduction*, 1st ed. Delft: Delft University Press, 2000.

[7] ——, *Testing theory; an introduction*, 1st ed. Delft: Delft University Press, 2000.

[8] D. Odijk, F. Kenselaar, and R. Hanssen, "Integration of leveling and InSAR data for land subsidence monitoring," vol. Submitted, 2004.

[9] R. Hanssen, "Stochastic modeling of time series radar interferometry," in *International Geoscience and Remote Sensing Symposium, Anchorage, Alaska, 20–24 September 2004*, 2004, pp. cdrom, 4 pages.