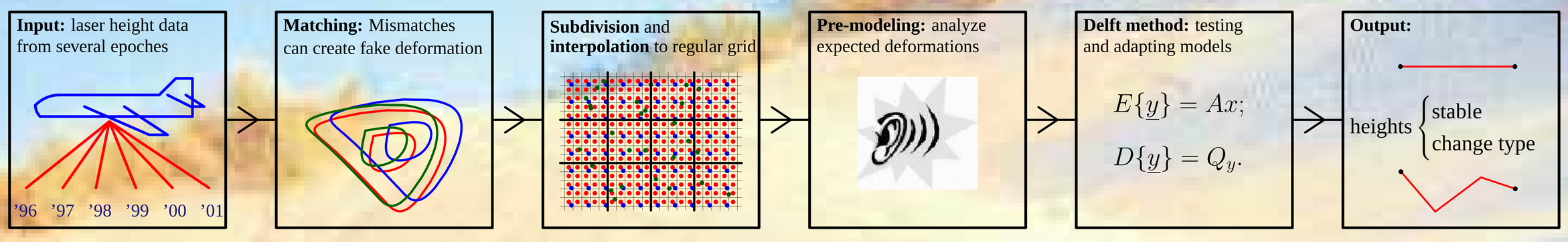
Eolian deformation detection and modeling using time series of airborne laser data

Applied on a dune section on the south-west of Texel, the Netherlands.

Time series analysis.



Single position linear modeling.

Assume that for every position (x, y) a full vector $\underline{h} \in \mathbb{R}^m$ of observed heights and a covariance matrix $Q_h \in \mathcal{M}(m, m)$ are given. We look for a linear model $A \in \mathcal{M}(m, n)$ such that

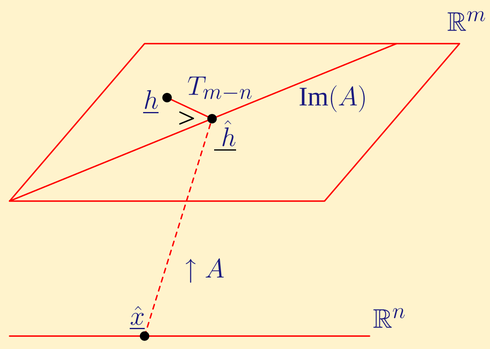
$$\underline{h} = \begin{pmatrix} h_{96} \\ h_{97} \\ \vdots \\ h_{01} \end{pmatrix}, \qquad H_0 : E\{\underline{h}\} = Ax, \qquad D(\underline{h}) = Q_h,$$

where $x \in \mathbb{R}^n$ denotes the vector of model parameters. The candidate model A needs to be tested for validity, while adjusting the observations.

Adjusting and Testing.

The estimator for the model parameters:

$$\hat{\underline{x}} = (A^T Q_h^{-1} A)^{-1} A^T Q_h^{-1} \underline{h}.$$

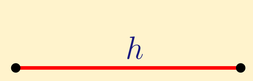
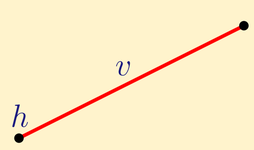
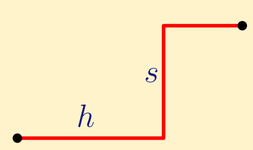


Let $\hat{\underline{e}} = \underline{h} - \hat{\underline{h}}$ be the vector of least squares residuals. The test statistic for the Overall Model Test:

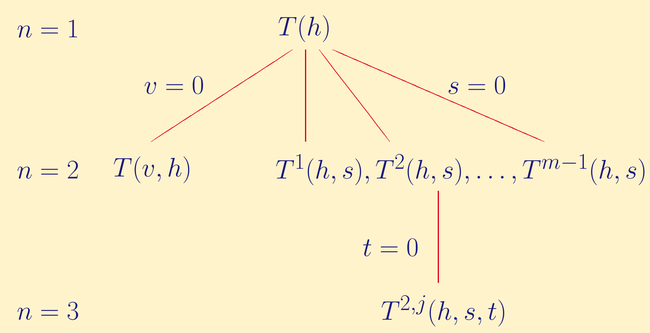
$$T_{m-n} = \hat{\underline{e}}^T Q_h^{-1} \hat{\underline{e}},$$

T_{m-n} has a $\chi^2(m-n, 0)$ distribution if the null hypothesis is true. We use $\alpha = .005$ as level of signi£cance, given the critical values $\kappa_\alpha \sim 14.9$ for $m = 6, n = 2$ and $\kappa_\alpha \sim 16.8$ for $m = 6, n = 1$. A test is accepted if $T_{m-n} < \kappa_\alpha$.

Model alternatives.

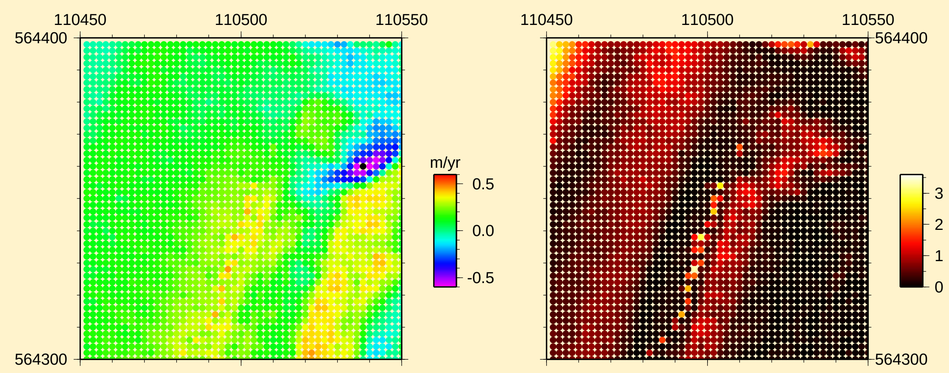
Stability test $T(h)$	Constant velocity test $T(v, h)$	Instant. deformation test $T^i(h, s)$
		
$A = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \quad x = (h);$	$A = \begin{pmatrix} t_1 & 1 \\ t_2 & 1 \\ \vdots & \vdots \\ t_m & 1 \end{pmatrix}, \quad x = \begin{pmatrix} v \\ h \end{pmatrix};$	$A = \begin{pmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{pmatrix}, \quad x = \begin{pmatrix} h \\ s \end{pmatrix}.$

Testing procedure.



For every position, the testing procedure starts at $n = 1$ with the stability test. If this test is not accepted, we continue with $n = 2$. All $n = 2$ test statistics are computed If a position fails every $n = 2$ test, it continues with $n = 3$. In the other case the model which produces the lowest test statistic is chosen as the appropriate model.

Example: best constant velocities and least squares error for $100m$ square.



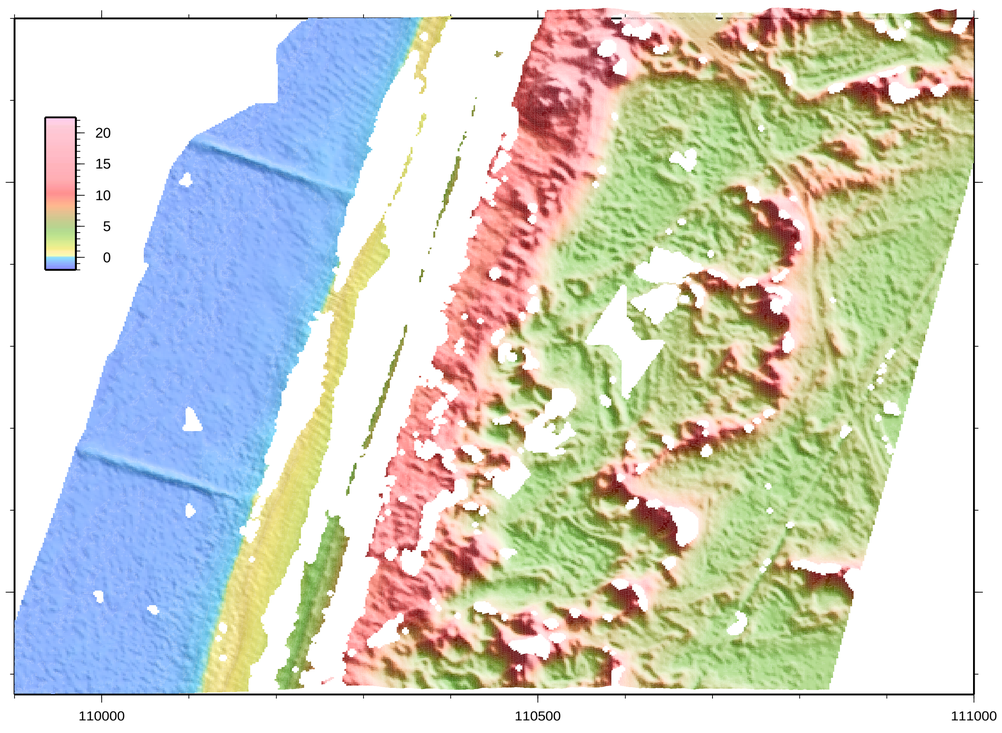
Results from testing procedure.

description	test	# positions
no interpolation		162 269
stable positions	$T(h)$	549 442
constant speed	$T(v, h)$	25 114
inst. change '96–'97	$T^{96}(h, s)$	5 519
inst. change '97–'98	$T^{97}(h, s)$	1 828
inst. change '98–'99	$T^{98}(h, s)$	3 436
inst. change '99–'00	$T^{99}(h, s)$	26 051
inst. change '00-'01	$T^{00}(h, s)$	3 431
no test passed		5 410
total		782 500

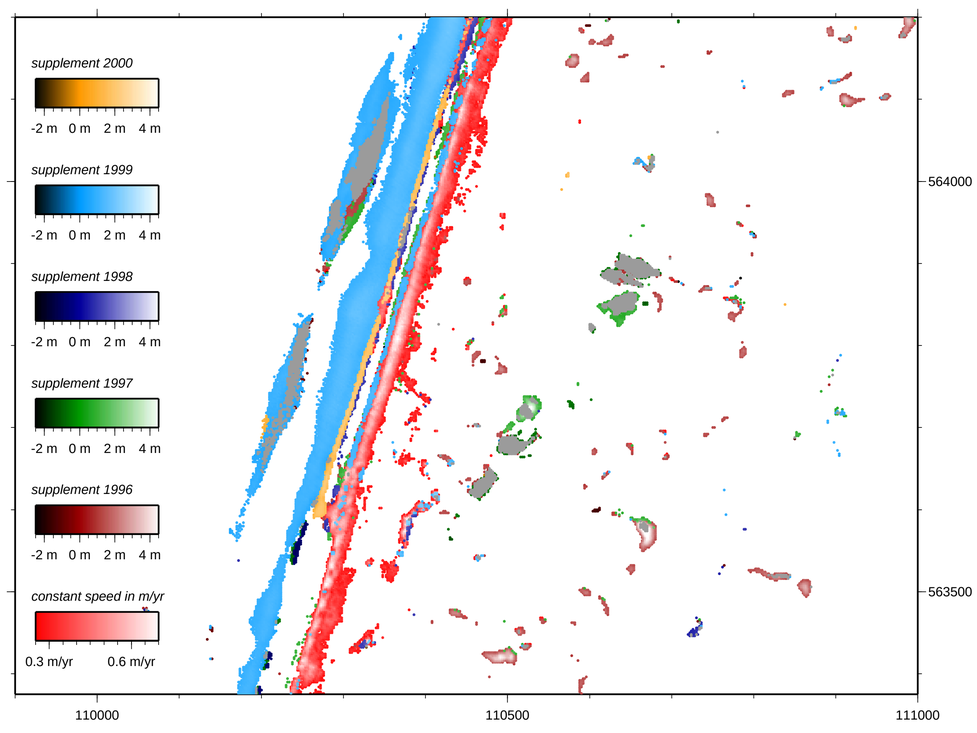
Further research.

- Checking results with Texel ground truth.
- Adding spatial correlation by developing multi-position tests.
- Improving the current setup: analyze the in¤uence of the choice of interpolation procedure, the covariance matrix Q_h and the level of significance α; add processing of positions with one height missing; add more tests/physical models.
- Analyzing other data sets.

Positions with stable height.



Positions where the height has changed.



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