Eolian deformation detection and modeling using time series of airborne laser data

Applied on a dune section on the south-west of Texel, the Netherlands.

Time series analysis.

- **Input**: laser height data from several epoches
- **Matching**: Mismatches can create fake deformation
- **Subdivision and interpolation** to regular grid
- **Pre-modelling**: analyze expected deformations
- **Delft method**: testing and adapting models

Output:

- heights stable
- change type

Single position linear modeling.

Assume that for every position \((x, y)\) a full vector \(h(x, y)\) of observed heights and a covariance matrix \(Q_h(x, y)\) are given. We look for a linear model \(A \in \mathcal{M}(m, n)\) such that

\[
\mathbf{h} = (\mathbf{A}^T \mathbf{Q}_h^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{Q}_h^{-1} \mathbf{h},
\]

where \(x \in \mathbb{R}^n\) denotes the vector of model parameters. The candidate model \(A\) needs to be tested for validity, while adjusting the observations.

Adjusting and Testing.

The estimator for the model parameters:

\[
\hat{h} = (\hat{A}^T \mathbf{Q}_h^{-1} \hat{A})^{-1} \hat{A}^T \mathbf{Q}_h^{-1} \mathbf{h}.
\]

Let \(\mathbf{e} = \mathbf{h} - \hat{h}\) be the vector of least squares residuals. The test statistic for the Overall Model Test:

\[
T_{\mathbf{e} = 0} = \mathbf{e}^T \mathbf{Q}_h^{-1} \mathbf{e} \sim F(m, n).
\]

\(T_{\mathbf{e} = 0}\) has a \(\chi^2(m - n, 0)\) distribution if the null hypothesis is true. We use \(\alpha = .05\) as level of significance, given the critical values \(s_{05} \sim 14.0\) for \(m = n = 2\) and \(s_{05} \sim 16.8\) for \(m = n = 6\). A test is accepted if \(T_{\mathbf{e} = 0} < s_{05}\).

Model alternatives.

- **Stability test** \(T(v)\)
- **Constant velocity test** \(T(h, v)\)
- **Instant deformation test** \(T(h, s)\)

\[
\mathbf{A} = \begin{pmatrix}
1 & 0 \\
1 & 1 \\
\end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix}
1 & 1 \\
1 & 1 \\
\end{pmatrix}
\]

\[
\hat{v} = \frac{1}{\mathbf{D}} \mathbf{h}, \quad \hat{h} = \frac{1}{\mathbf{D}} \mathbf{h}, \quad \hat{s} = \frac{1}{\mathbf{D}} \mathbf{h}.
\]

Example: best constant velocities and least squares error for 100m² square.

Results from testing procedure.

- **Description**
- **Test**
- **# positions**

<table>
<thead>
<tr>
<th>Interpolation</th>
<th>No interpolation</th>
<th>Constant</th>
<th>Instant change '96–'97</th>
<th>Instant change '97–'98</th>
<th>Instant change '98–'99</th>
<th>Instant change '99–'00</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stable</strong></td>
<td>564300</td>
<td>564400</td>
<td>563500</td>
<td>564000</td>
<td>564200</td>
<td>564300</td>
<td>564300</td>
</tr>
<tr>
<td><strong>Interpolated</strong></td>
<td>162 269</td>
<td>25 114</td>
<td>5 128</td>
<td>3 436</td>
<td>26 051</td>
<td>5 128</td>
<td>5 128</td>
</tr>
</tbody>
</table>

Further research.

- Checking results with Texel ground truth.
- Adding spatial correlation by developing multi-position tests.
- Improving the current setup: analyze the influence of the choice of interpolation procedure, the covariance matrix \(Q_h\) and the level of significance \(\alpha\); add processing of positions with one height missing; add more tests/physical models.
- Analyzing other data sets.

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