Abstract.
The Dutch part of the North Sea is mapped by two authorities. Both authorities predict the behavior of the sea floor by analyzing time series of bathymetric data. The Hydrographic Service determines parameters for sand wave dynamics from an area, based on deformation analysis. The North Sea Directorate uses a multivariate state space modelling approach to estimate, update and extrapolate trends per grid point. Here, both approaches are described and combined in order to obtain a prediction method that incorporates a sand wave propagation model. The new method is tested on two data sets of different characteristics.

1. Introduction

The Dutch part of the North Sea is being mapped by two authorities, the Hydrographic Service of the Royal Netherlands Navy and the North Sea Directorate, part of the Directorate-General of Public Works and Water Management. Monitoring the depth in the Southern North Sea is essential, because it is heavily used by shipping, but shallow. To ensure that main ports, such as Rotterdam, remain safely accessible, reliable depth information is needed. However, the depth is not constant in time, due to e.g. the development of sand waves. These are regular wave patterns, having amplitudes of several meters. These sand waves tend to grow and migrate, which may have consequences for the navigable channel depth.

As it is expensive to survey large parts of the sea on a regular basis, insight in the dynamic behaviour of the sea floor is required. Both authorities have developed their own method to monitor the sea floor by using time series of bathymetric echo sounding data. In this article, parts of both methods will be described and combined to a new method that benefits from both.

The Hydrographic Service is responsible for the production and upkeep of nautical publications, such as nautical charts. In order to make a more efficient planning of the bathymetric surveys used for the production of these charts, insight in the dynamics of the sea floor is needed. Therefore a method is developed, [1], for analyzing time series, based on geodetic deformation analysis. The core of this method is a testing procedure to determine if the sea floor is static or contains some kind of dynamics. Two kinds of dynamics are considered: outlying surveys and linear trends. This procedure can be applied at several scales, for single grid-points or for a
whole area. The point-wise test results can be used for predictions. Furthermore, sand wave parameters are estimated within the area test. This method proves to work very good for the detection of dynamics, but the prediction results are not satisfying.

The North Sea Directorate is responsible for maintenance of the sea channels, like the Euro Channel to the port of Rotterdam. For this channel, a nautical guaranteed depth is defined: when the channel becomes too shallow it has to be dredged. To predict the moment when the depth in the channel will rise above a critical depth, a trend analysis model has been developed, [2], based on a state space model and Kalman filtering. In this method a linear growth model is attributed to every point of a fixed grid. This growth model consists of a depth part and a linear part which are updated when new measurements become available. Except for the actual depth and trend values, accuracy measures for these values are determined as well. These depths and trends, together with their accuracy, are used to make a prediction for the future sea depth. This method performs well for predictions up to five years ahead and also produces calibrated predictive uncertainty measures.

Both authorities have developed a grid-point wise prediction setup, but in both cases more complicated dynamics, like propagating sand waves, are not incorporated in the grid-point wise predictions. To overcome this shortcoming, a combination of both methods is proposed. First a deformation analysis is applied to detect outlying surveys and determine sand wave migration parameters. The detected outliers are used to correct the input data of the state space model. Furthermore, the model filter is extended with a local testing procedure and a sand wave propagation model, based on the parameters found in the deformation analysis.

This combined method is tested on two data sets of different characteristics. On the data set containing an obvious regular sand wave it proves to work well. The benefit of the new method on the other data set is less clear, because the mapped area is less dynamic and the sand waves are less regular. This demonstrates the importance of an efficient model snooping procedure that should select the most appropriate model for describing the morphology of a local sea floor surface and its dynamics. It should be noticed however that in e.g. the area monitored by the North Sea Directorate sand waves are either reported to move not at all, or at propagation velocities of at most 2 m a year. Even in the latter case the improvements as obtained from the combined approach will still be limited.

In Section 2 the combined method is described in some detail, in Section 3 the prediction results on real data are presented, followed by conclusions in Section 4.

2. Prediction Methodology

In this section a method is described that allows for fitting a propagating sinusoidal sand wave model onto a regular grid of sea floor depth data for the purpose of predicting future grid-point wise depth values. First, an example is given that demonstrates the benefit of such an approach. Then, it is shown how to obtain initial sand wave parameters, and how to integrate these parameters into a state space model. The model can be used on the one hand to update the sand wave model with new observations, and on the other hand to obtain predictions of expected dynamics.

2.1 A simulated example
Consider a simulated propagating sand wave with an amplitude of 1m and a wave length of 300 m. Every year, the crest is shifted 10m to the right. In Fig. 1, left, the sand wave positions in the years 2001, 2002, 2003 and 2004 are shown. On the right, the analysis results of a deformation analysis per grid-point are given. Points on the left become deeper, while points on the right become shallower. From this simulation it is obvious that sand wave dynamics with a local growth model will result in a prediction accuracy that decreases with increasing sand wave propagation. Such predictions can be improved by incorporating a spatial area based sand wave model into the grid-point wise prediction method as this allows for modelling the sand wave propagation. The essential extension is that in this case a global model is used for predicting local dynamics.

Fig. 1: Left: profile of a simulated migrating sand wave. Right: depth predictions.

2.2 Experimental covariance analysis

The spatial continuity between neighbouring sea floor depth observations is traced by means of an experimental covariance analysis, under the assumption that the depth signal is stationary, [3]. This assumption states that the expected covariance between two depth observations is independent from the location of the observations in the signal. The covariance function is estimated by multiplying for any pair of depth observations $d_i$ and $d_j$, the observation wise differences with the mean depth $\mu$. The resulting numbers $c_{ij} = (d_i - \mu) \cdot (d_j - \mu)$ are now grouped with respect to the vector $p_i - p_j$ that denotes the difference between the observation locations $p_i$ and $p_j$. By averaging groups of experimental covariances with similar difference vectors, an approximate, direction dependent experimental covariance function is obtained.

Covariance functions are used to describe the redundancy between depth observations in geostatistical interpolation methods, but in this case they are applied for obtaining an estimate of the average direction and length of sand waves, compare [1]. The direction of highest variability, $\alpha_P$, is assumed to correspond with the propagation direction of the sand waves and can be estimated by analyzing the gradients of the depth data. Along this direction, covariance values will be small at distances that correspond to half the sand wave length, but will be larger again at one sand wave length distance $L$. This parameter $L$ is estimated from the experimental covariance function by taking an optimal fit parameter obtained from fitting a suited smooth function to the experimental covariance function.
2.3 Static sea floor model

After determining a suitable model of the sea floor, based on one epoch of data, the dynamics of the sea floor are determined in a deformation analysis procedure. For modelling the sea floor, three scenarios are considered, the sea floor as a flat plane, the sea floor as a sloping plane and the sea floor as a sloping plane with sand waves. The latter model expresses the depth \( d_p \) at point \( p \) with horizontal coordinates \((x_p, y_p)\) as

\[
E \{ d_p \} = \{1, x_p, y_p, \cos(2 \pi x_p / L) - \sin(2 \pi x_p / L)\} \cdot \{d, \alpha_x, \alpha_y, u, v\}^T.
\]

with \( \alpha_x \) and \( \alpha_y \) the mean slopes in the \( x \)- and \( y \)-direction, \( d \) the mean depth of the area and \( L \) the wave length of the sand waves. The wave length \( L \) is determined by a covariance analysis as indicated above. The remaining parameters, \( u = A \cos \phi \) and \( v = A \sin \phi \) describe a one-dimensional wave, with \( A \) the wave amplitude, and \( \phi \) its initial phase via

\[
A \cdot \cos(2 \pi x_p / L + \phi ) = \cos(2 \pi x_p / L) \cdot u - \sin(2 \pi x_p / L) \cdot v,
\]

In this way, the depth \( d_p \) at position \( p \) depends linearly on the estimation parameters \((d, \alpha_x, \alpha_y, u, v)\). By the back substitution \( A = \sqrt{(u^2 + v^2)} \) and \( \phi = \arctan v/u \), the sand wave amplitude \( A \), and sand wave phase, \( \phi \), are recovered from the estimated parameters \( u \) and \( v \).

Selection of the most likely sea floor model is made by a testing procedure, ([4, 1]). In such a procedure, the distance of the observations to an alternative model, that is, an extension of the current model, is determined in an adjustment step. The different distances to the alternative models are compared in a hypothesis snooping procedure that incorporates the number of parameters in each model, compared to the number of observations. Subsequently, the model is extended with the most relevant extension. This procedure continues until none of the remaining alternatives significantly improves the fit of the model to the data anymore.

2.4 Deformation analysis

Now it is assumed that the sloping plane with sand waves is tested to give the most adequate description of the sea floor as sampled by the observations in the first epoch, observed at \( t_0 \). The other epochs, \( k \), observed at \( t_k \), are used to determine the dynamics of this sea floor model. Different scenarios for the sea floor behaviour in the area of study are compared: stability, a single outlying survey, trends in depth and slopes, and in amplitude and phase of the sand waves. The so-called null hypothesis states that the sea floor is static through time. If the null hypothesis is probable, that is, if the corresponding test statistic is below the critical value, the null hypothesis is accepted and the sea floor is considered static.

If the null hypothesis is rejected, the alternative hypotheses are considered: for all alternatives the quotient between the hypothesis wise test statistic (distance observations to model) and the critical value is determined. The alternative that has the largest quotient is selected. In case of the detection of a single outlying survey, all observations of the outlying epoch are removed and the procedure is repeated. As an example, the dynamic model of a trend in the sand wave is constructed by extending the current sea floor model with a term
\[
\cos(2 \cdot \pi \cdot x / L) \cdot (t_k - t_0) \cdot \Delta u / \Delta t - \sin(2 \cdot \pi \cdot x / L) \cdot (t_k - t_0) \cdot \Delta v / \Delta t
\]  

(3)

Adjusting the observations into this model will result in wave propagation parameters \(\Delta u / \Delta t\) and \(\Delta v / \Delta t\) that together describe a change in amplitude, \(\Delta A\), and phase, \(\Delta \phi\), of the sand waves in the study area.

2.5 Kalman filtering and the state space sand wave model

The dynamic area-wise sand wave model can be incorporated in a grid-point wise state space model. This model can be used to obtain depth prediction at grid points at arbitrary time \(t\). The deterministic part of the model, [5, 2], consists of a state vector \(x_k\) containing model parameter values at time \(k\) and a transition matrix \(\Phi_{t,k}\) that describes the change in model parameter values \(x_k\) at time \(k\) to the values \(x_t\) at time \(t\), that is, \(x_t = \Phi_{t,k} \cdot x_k\). This step is called time update, as the state of the parameter values in the past is updated to current time, say. It is also possible that new measurements become available. Then a measurement update step is required that adjusts the (time updated) parameter values by incorporating the new observations at that time using the Kalman filter.

Besides a deterministic part, the state space sand wave model also contains a stochastic part. The latter part separately addresses the accuracy of the input observations and the growing uncertainty of the state space model predictions when evolving to future time steps.

Another way of error handling is by means of the so-called DIA procedure, [5]. DIA stands for Detection, Identification and Adaption. In the detection step it is tested whether the observations fit the null hypothesis model. If the distance between the observations and model is too large, the cause of the large distance has to be identified in the identification step. In this step several model extensions (alternative hypotheses) are considered, for example extensions that model an outlier in the, say, \(k\)-th observation. If e.g. an outlier is identified, the final step of the DIA procedure is entered in which the state vector is adapted in such way that the reported outlier does not influence the predictions anymore.

In order to express the above dynamic sand wave model in a state space format, the transition matrix \(\Phi_{t,k}\) and the state vector must be expressed in terms of the dynamic sand wave model of Eq. 1. The sand wave can only be modelled if sufficient spatial information is available at every Kalman filter update step. Therefore the model maintains local depths \(d_i\), local depth trends \(\Delta d_i / \Delta t\) and the global sand wave parameter values \(u, v\) and \(\Delta u\) and \(\Delta v\) in a local grid around the prediction location. The positions in this grid are labelled from \(i=1\) to \(i=J\). The state vector is given by

\[
x = (d_1, \cdots, d_J, \Delta d_i / \Delta t, \cdots, \Delta d_j / \Delta t, u, \Delta u / \Delta t, v, \Delta v / \Delta t).
\]  

(4)

The transition matrix reflects the change in the state vector according to the dynamic model of above and has, with \(A' = (A_0 + \Delta A) / A_0\) and \(Id_{JxJ}\) the identity matrix of size \(J\), the shape
A description of the full state space model for sea floor depth predictions with a state vector consisting of grid-point wise depths and linear depth changes is described elsewhere, [2]. The model presented here extends this approach, and maintains an area wide spatial model of the sea floor.

2.6 Final prediction algorithm

The steps leading to a grid-point wise prediction, as explained in some detail above, are summarized in the following algorithm.

**Input:** Data grid available in K epochs. At every grid point a depth value and a depth variance is available.

1. Determine sand wave length and sand wave direction from experimental spatial covariance analysis.
2. Determine sand wave velocity, sand wave amplitude and outlying surveys from a deformation analysis of all K epochs. Remove outlying surveys if found.
3. Run a state space model on the remaining data. At every Kalman filter update step, grid-point wise outliers are identified and eliminated by a DIA procedure.
4. Perform state space evolution step for a prediction at an arbitrary future moment.

**Output:** grid-point wise predictions.

3. Prediction algorithm results.

Here the results of the new algorithm are given on two data sets. The first data set, A, features a regular sand wave. The morphology in the second data set, B, is far less regular however.

3.1 Data set A

Fig. 2., bottom left, shows the depth of area A in 1991. Area A is located in the Euro Channel, indicated by the right most rectangle in the North Sea map in Fig. 2. The presence of a sand wave in this data set is obvious. This area of 330 x 330 m has been monitored by Multi Beam Echo Sounding in all years between 1991 and 2001, except for 1992 and 1998. The data are available on a 5m grid.

From the experimental covariance function a sand wave length of 225 m was found and a sand wave orientation of 47 degrees East of North. Least squares adjustment to the dynamic sand wave model as described in Section 2, gives a sand wave amplitude of about 1.4 m and sand wave propagation velocity of 1.6 m/year. The amplitude growth is negligible. Two profiles, illustrating the sinusoidal sand wave model, are given in Fig. 2, top left and right. From the depth plot it can be seen...
that the sand wave is curved. Therefore the modelled wave fits better in the middle (top left) than at the edges (top right).

Once all initial sand wave parameters were estimated, the state space sand wave model was run sequentially on the epochs of gridded observations. On the bottom right of Fig. 2 the one year prediction for individual grid points is given. That is, the differences between the observations of the last survey and the predictions for one year ahead are displayed. Clearly, the motion of the sand wave in North-East direction is visualized by the pointed triangles, indicating upwards resp. downwards movements of at least 5 cm. A validation of the prediction results is obtained by comparing predictions for e.g. 2001 to the actual grid wise observations. The mean absolute difference between prediction and observation is 12 cm, with maximum values of +45 cm and -12 cm.

![Fig. 2. Data set A. Profiles of the modelled and the real sand wave are shown in the top left and right. The location of the profiles is indicated in the depth map at the bottom left. The position of this data set (right rectangle) and of data set B is indicated in the map at the top. At the bottom right, predictions are given based on the dynamic state space sand wave model.](image)

### 3.2 Data set B

The second data set that was processed is data set B, see Fig.3. This data set represents a part of the so-called `selected track', which consists of the seaway to the port of Rotterdam and the anchorages. The location of the selected track is indicated by the left most rectangle in the North Sea map in Fig. 2. This data set is obtained by a single beam echo sounder in the years 1991, 1995, 2000, 2001, 2002...
and 2003. This is a difficult area: although the area is clearly not flat, it does not have a regular sand wave either.

In this case a prediction is made without modelling a sand wave. Differences between state space predictions and measured depths are within -0.47 m and +0.76 m. No regular pattern or strong dynamics can be seen from the one year prediction map shown in Fig. 3, right. It would only make sense to estimate the regular sinusoidal sand wave model, if the data set is subdivided into segments containing only sand waves with similar orientation. As in this case the area is not very dynamic anyway, gain, in the sense of more reliable depth predictions, would be limited.

Fig. 3. Left: interpolated depth map of data set B. Right: One year ahead state space model prediction without estimation of a sinusoidal sand wave model.

4. CONCLUSIONS

A method has been proposed to incorporate an area wide morphological sea floor model in a state space Kalman filter model for the purpose of grid-point wise change predictions. In this particular case a propagating sinusoidal sand wave was modelled.

Test results indicate that on regularly shaped and moving sand wave areas more reliable and therefore more cost-effective predictions are obtained by using this state space sand wave model.

Dynamic areas with an irregular morphology can be automatically reported by the deformation analysis component as fitting badly to any tested dynamical model. In such a case the area could be segmented in more regular sub-areas.

REFERENCES