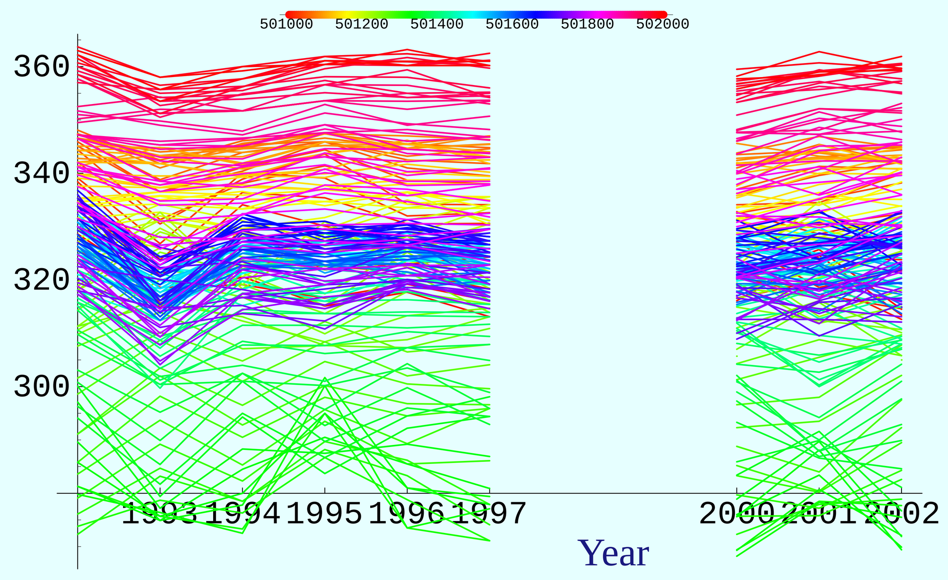
Parameter estimation and deformation analysis of sand waves and mega ripples.

Introduction.

Given are nine years, (1992,..., 1997, 2000, 2001, 2002), of Multibeam Echosounding data, interpolated to a $5m \times 5m$ grid of a seabottom area of 1km^2 in the Euro channel approach. We want to model possible ongoing deformation during this period and obtain parameters for sandwaves and mega ripples.

Depth in dm.



Depths during the years at positions along a horizontal profile. Larger variations occur at lower depths.

Vertical deformation analysis at single grid points.

Linear vertical deformation at a single grid point can be traced by classical deformation analysis: a linear model A with, in this case, $m = 9$ years of depth observations d_i and a n-parameter vector x is adjusted and tested on it self, or against other models:

$$\underline{d} = \begin{pmatrix} d_{92} \\ \vdots \\ d_{97} \\ d_{00} \\ \vdots \\ d_{02} \end{pmatrix}, \qquad H_0 : E\{\underline{d}\} = Ax, \qquad D(\underline{d}) = Q_d.$$

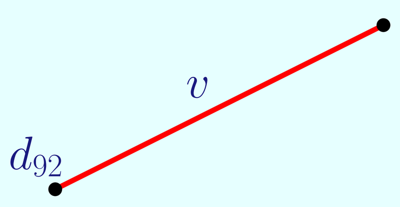
The adjustment step consists of projecting the observations into the model space, while in the testing step the test statistic, that is, the weighted sum of residues between the observed depths and the adjusted depths, is compared to a critical value.

Example: constant velocity model.

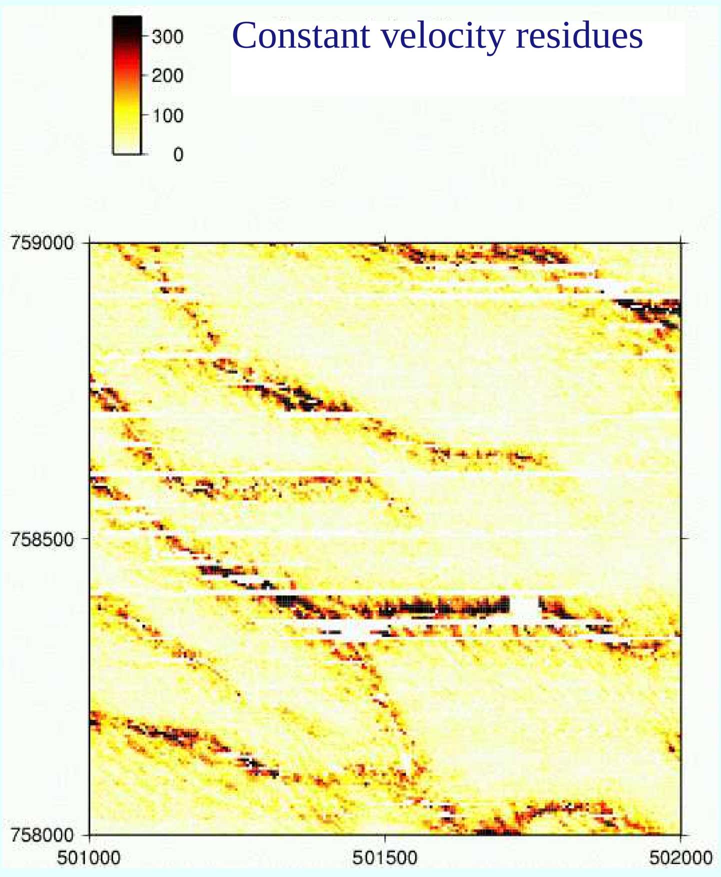
In the constant velocity model, the dephth $d(t)$ at time t is a linear function of time. That is

$$d(t) = d_{92} + v \cdot t,$$

where d_{92} denotes the depth at the beginning of the measurements and v denotes the deformation per year.



$$A = \begin{pmatrix} t_{92} & 1 \\ \vdots & \vdots \\ t_{97} & 1 \\ t_{00} & 1 \\ t_{01} & 1 \\ t_{02} & 1 \end{pmatrix}, \quad x = \begin{pmatrix} v \\ d_{92} \end{pmatrix};$$

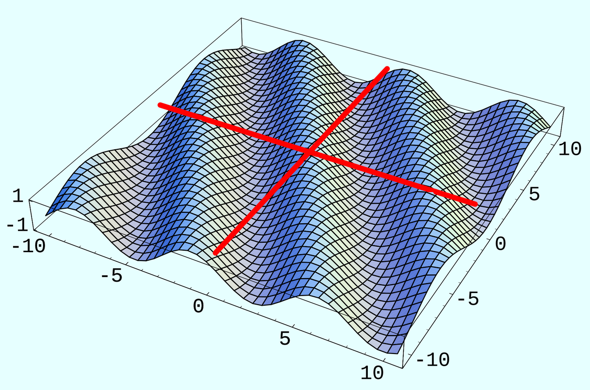


On the right the residues of the constant velocity test are shown. There is a a clear correlation between the location of the sandwave crests and the higher residue values. Similar results are obtained if one determines the residues for a constant depth model.

More complicated alternative: local-global sandwave model.

To give a local description of the motion of the mega ripples we use the equation that describes a plane wave that is propagating in one direction:

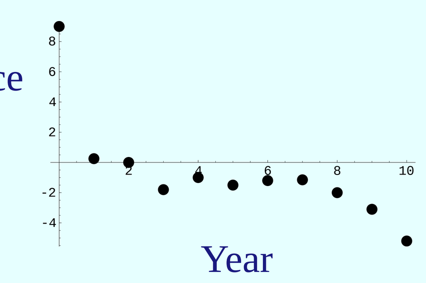
$$\psi(x, y, A, k, \phi_0, \theta, v, t, d) := A \sin[k(x\cos\theta + y\sin\theta) - vt + \phi_0] + d \qquad (1)$$

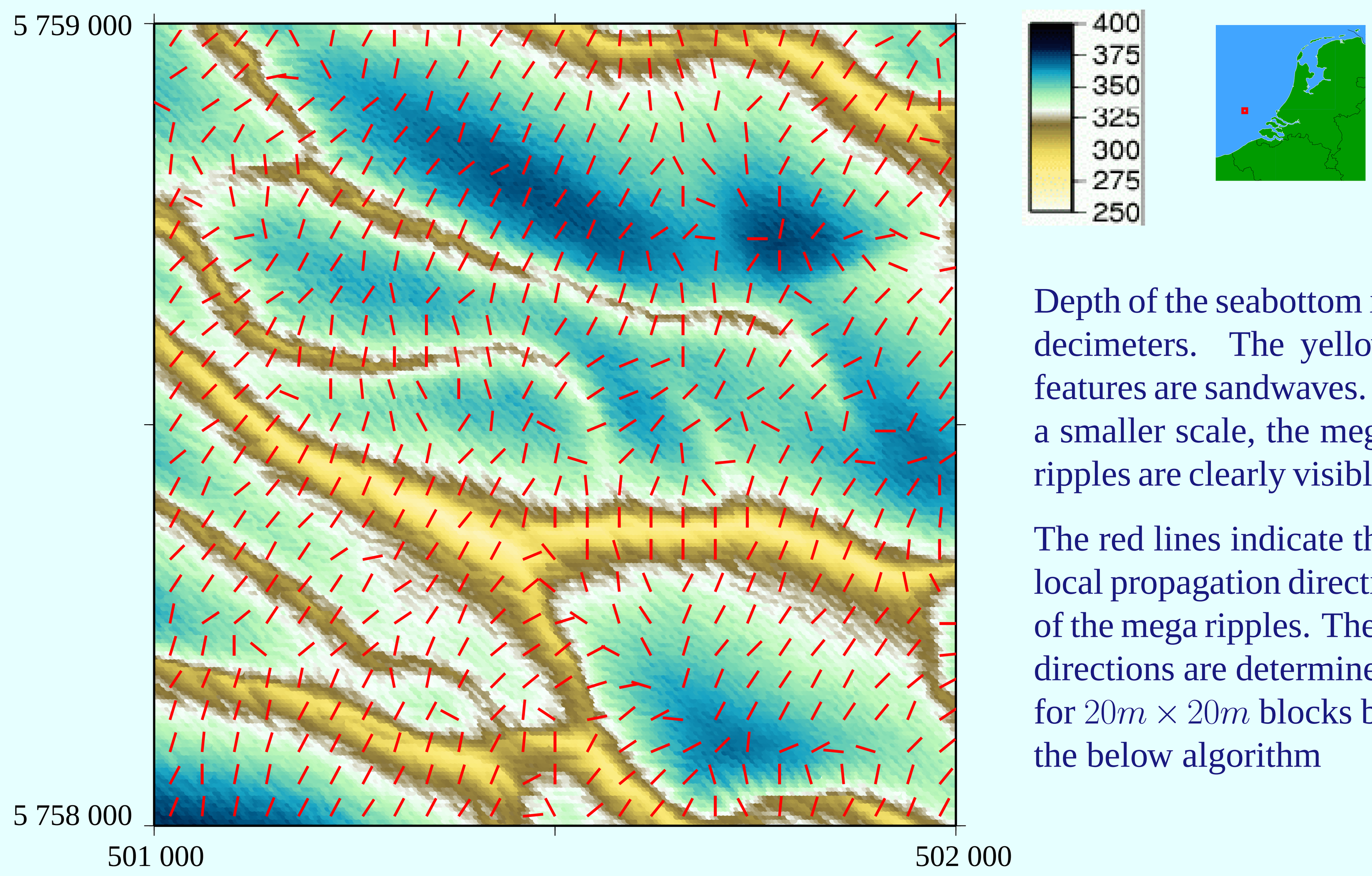


- A Amplitude
- k wave number
- ϕ_0 initial phase
- θ angle between propagation direction of the wave and horizontal x-axis
- t time
- v wave velocity
- d average depth

Wave number, velocity and average depth

We determine the wave number of the mega ripples simply by counting the number of crests. The average depth can be obtained by the linear tests of above. The covariogram of variation of depth in time at all grid points shows periodic correlation giving an indication for the propagation velocity of the mega ripples.





Depth of the seabottom in decimeters. The yellow features are sandwaves. On a smaller scale, the mega ripples are clearly visible.

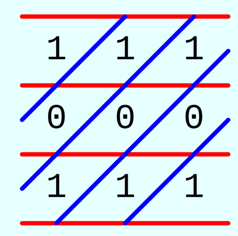
The red lines indicate the local propagation direction of the mega ripples. These directions are determined for $20m \times 20m$ blocks by the below algorithm

Local mega ripple direction

For determining the mega ripple direction, we use the algoritm of below. The mean idea of the algoritm is that we expect *low* local depth variation parallel to the mega ripple crests and *high* variation in the propagation direction of the mega ripples.

Low variation parallel to the red lines

High variation parallel to the blue lines



0	1/2	1/3
1/2	2/3	1/2
1/3	1/2	0

On the right, the indiviual variations in the direction parallel to the blue lines are given

Algorithm: local, ($20m \times 20m$), mega ripple direction.
Input: regular $n \times n$ block of *xyz*-data.
Output: direction θ of lowest variation in the block.

1: **for** $\theta \in -\pi/2, -\pi/2 + \pi/(2n), \dots, \pi/2 - \pi/(2n)$ **do**

2: Use $R_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ for rotating all (x, y) values in the block. Let y_- and y_+ be the minimal and maximal y-value *after* rotation. The strip width is defined as $w_\theta := (y_+ - y_-)/n$.

3: Divide the depths z into m horizontal strips

$$[y_-, \; y_- + w_\theta), \dots, [y_- + (m-1) \cdot w_\theta, \; y_- + m \cdot w_\theta),$$

according to their rotated y-value $R_\theta(y)$. Every depth belongs to exactly one strip S_k.

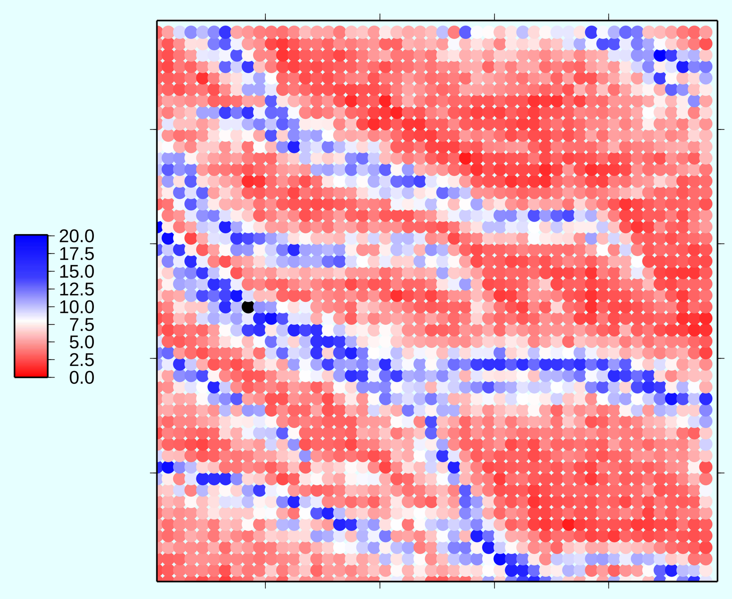
4: Suppose z_i belongs to strip S_k. Then the *individual variation* of z_i is defined as $v_\theta(z_i) = z_i - \mu_k$ where μ_k denotes the average of the depths in strip S_k.

5: The *overall variation* in direction θ is just the sum of squares of all individual variations: $V_\theta = \sum_i v_\theta(z_i)^2$.

6: **return** θ with minimal overall variation.

The results of the algorithm for $20m \times 20m$ blocks are shown in the figure above.

Local mega ripple amplitude.



Consider $20m \times 20m$ blocks of the 2002 data containing on average 15 depth observations. Let d_{max} and d_{min} be the maximal and minimal depth per block. The local amplitude is estimated by

$$A_{\text{block}} = (d_{\text{max}} - d_{\text{min}}) \cdot 0.518$$

The $0.518 = .5 \cdot 1.93$ is obtained by computing the expected maximal difference, 1.93 between 15 random sine numbers.

The estimated amplitudes of the mega ripples in decimeter. Note the strong correlation between the amplitude of the mega ripples and the location w.r.t. the sandwaves.

Conclusions and further research.

- The behaviour of mega ripples is strongly correlated with the location of sandwave crests.
- Vertical deformation analysis is less suited for seabottoms covered by sandwaves.
- The parameters found, should be validated by adjustment and testing theory.
- Further research in data validation of sandwave covered seabottom deformation should concentrate on the combination of horizontal and vertical deformation, that is, the flow of the sand.
- More attention should be paid to the combination of the driving forces, like the current, and the applied deformation models.
- Different deformation regimes should be separated, like sandwave motion and mega ripple motion.

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